Reconstruction Algorithms for Material Phase Contrast Tomography

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Problem

Computed Tomography (CT) is a widely used imaging technique most know the applicability in medical research, but also used within material science research for hard and soft tissues.

There is an increasing demand for higher resolution, better tissue distinction and lower X-ray dose which places higher demands on the two building blocks of CT:

- Data acquisition
- Mathematical Reconstruction

Phase Constrast Tomography (PCT) may give better resolution and tissue distinction for specific tissues, but introduces new demands data acquisition, modelling and reconstruction.

For this project Free Space Propagation, where no gratings or crystals are used, is the only PCT method investigated.

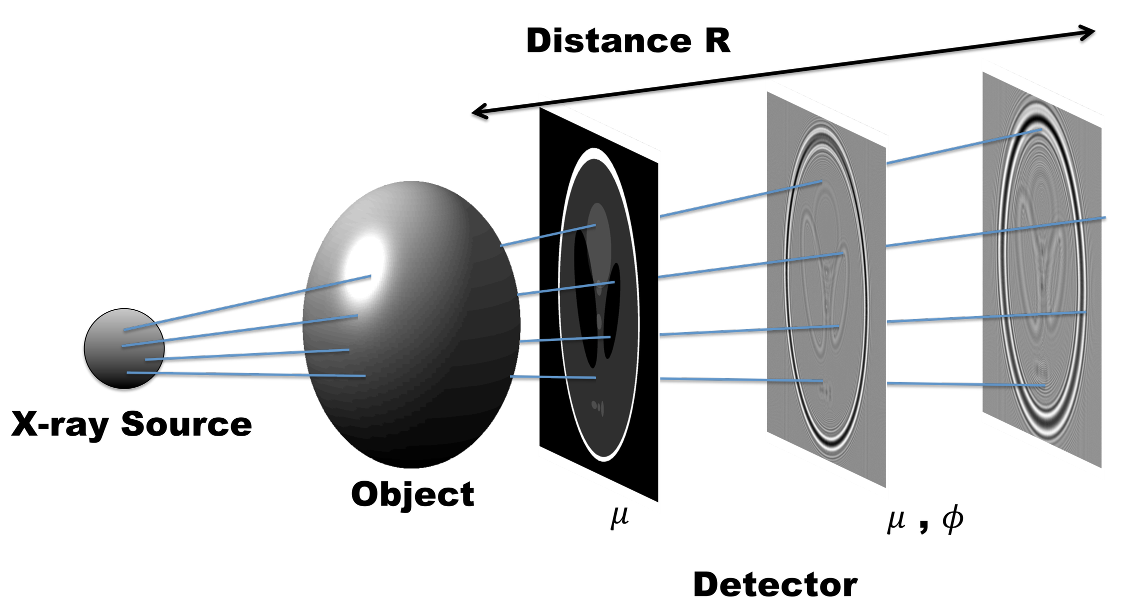
Absorption based CT

Absorption based CT (ACT) is based on the attenuation of X-rays though an object. The attenuation is modelled by Lambert-Beers law (here rearranged to get projection expression b):

$$b = \log\left(\frac{I_0}{I_L}\right) = \int_L f(\mathbf{x})\,\mathrm{d}\mathbf{x}$$

I_d is intensity at distance d from interaction with the object f, specified in coordinates $\mathbf{x}$.
The scanning geometry for this investigation is parallel beam projections.

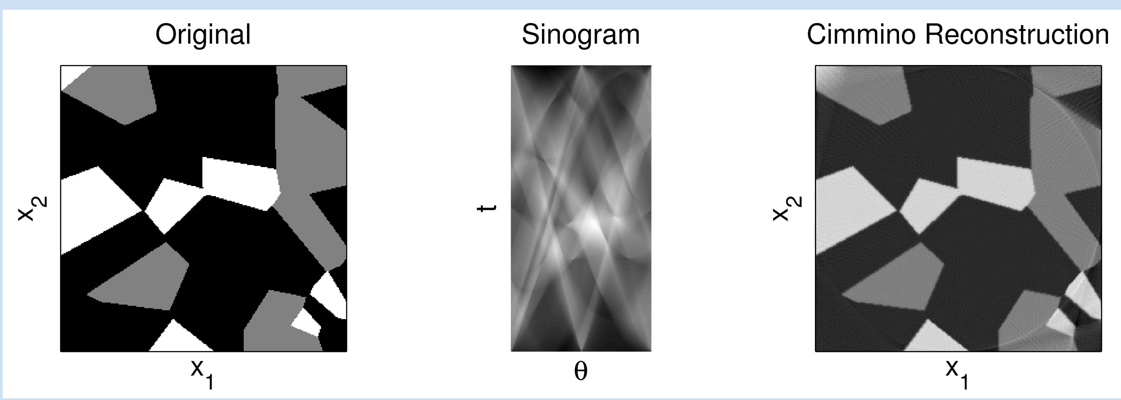
Data acquisition



Simple 3D sketch to show the difference in data acquisition between ACT and PCT. For PCT method called free space propagation the experimental set up is similar to ACT. When the detector is close to object, $R \sim 0$, only the attenuation coefficient μ is detected, where further down effects of both μ and phase shift ϕ is detected.

CT Reconstruction

CT reconstruction is an inverse problem which can be solved using fx analytical (Radon transform and FBP) or algebraic methods. The measurements are a series of projections from different angles called a *sinogram*. Sinogram and reconstruction using algebraic method *Cimmino*:



ACT reconstruction simulation example, where only μ is reconstructed from measurements without noise.

Phase Contrast Imaging

2D examples below, but can be generalized to 3D. Model for the refractive index of materials:

$$n(x_1, x_2) = 1 - \delta(x_1, x_2) + i\beta(x_1, x_2)$$

Indexes: refractive decrement δ, absorption β.

Projections for the specific angle $\theta = 0$:

$$\mu(x_1) = \frac{2\pi}{\lambda}\int \beta(x_1, x_2)\,\mathrm{d}x_2$$
$$\phi(x_1) = -\frac{2\pi}{\lambda}\int \delta(x_1, x_2)\,\mathrm{d}x_2$$

Wavelength λ of X-ray.

*T*ransmittance and *P*ropagation of X-ray wave:

$$T(x_1) = \mathrm{e}^{-\mu(x_1)+i\phi(x_1)}, \quad P_R(x_1) = \frac{1}{i\lambda R}\mathrm{e}^{\frac{i\pi}{\lambda R}x_1^2}$$

Model for intensity recorded at detector at distance R from object:

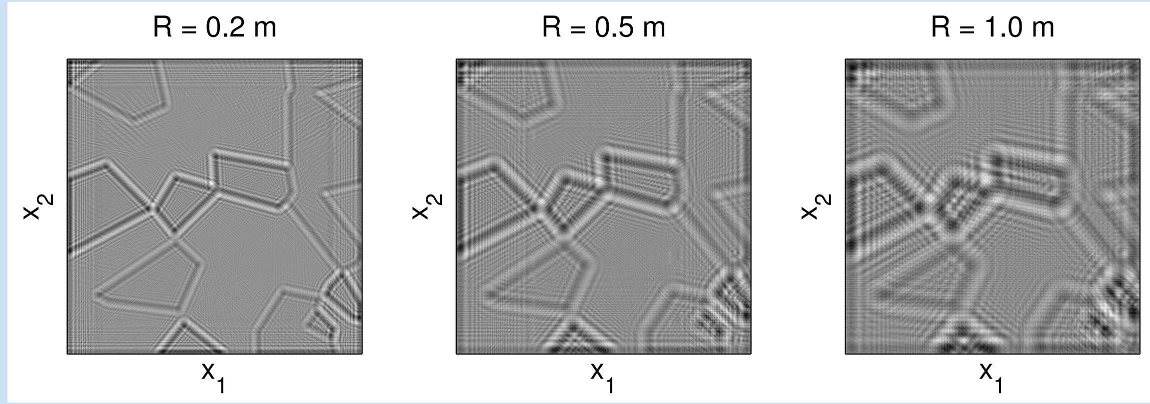
$$I_R(x) = |P_R(x_1) \star T(x_1)|^2$$

Convolution operator $\star$.

Measurements from different angles θ and a translations t perpendicular to the transmittance direction gives a measurement sinogram $I_R(\theta, t)$.

From the projections $I_R(\theta, t)$ the task is now to calculate back to get projections $\mu(\theta, t)$ and $\phi(\theta, t)$, this is called *phase retrieval*. Here the method Contrast Transfer Function (CTF) is used.

Reconstructions at different distances R without phase retrieval:



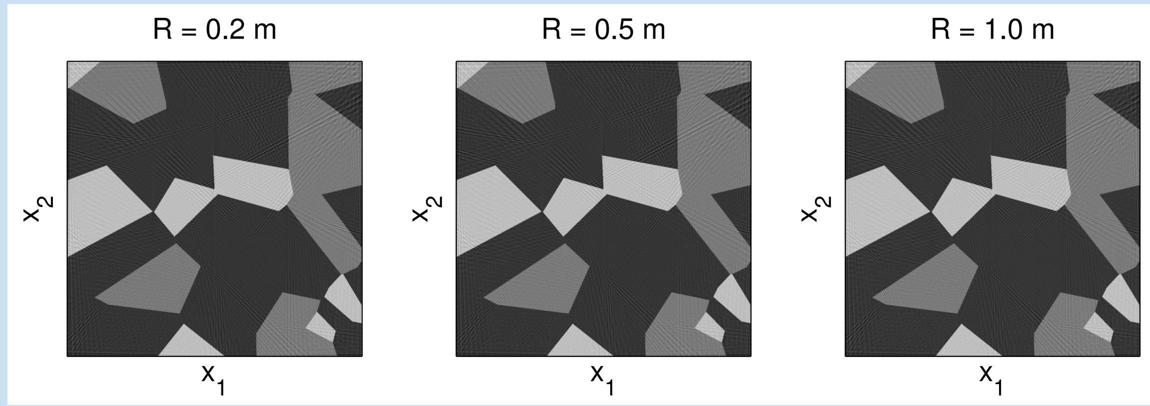
CTF

1. order Taylor-expansion of T, inserted into $I_R(x)$ gives a linear function in the Fourier domain with respect to the first order terms:

$$\hat{I}_R(\omega) = \delta_{dirac}(\omega) - 2\cos(\pi\lambda R|\omega|^2)\hat{\mu}(\omega) + 2\sin(\pi\lambda R|\omega|^2)\hat{\phi}(\omega)$$

Fourier transform $\hat{\cdot}$ with frequencies ω. This relation can be used to retrieve $\mu(\theta, t)$ and $\phi(\theta, t)$, though there are two unknowns $\hat{\mu}$ and $\hat{\phi}$ for each recorded intensity pixel $\hat{I_R}$. Therefore two (or more) distances ($R_1 \neq R_2$) or an assumption about proportionality between the unknowns (duality) is needed. After phase retrieval, reconstruction will give $\mu(x_1, x_2)$ and $\phi(x_1, x_2)$.

Duality example, with assumption $\sigma = \frac{\phi}{\mu}$, and no noise on data:



$\mu(x_1, x2)$ results for different distances.

Comparison of Tomography Methods

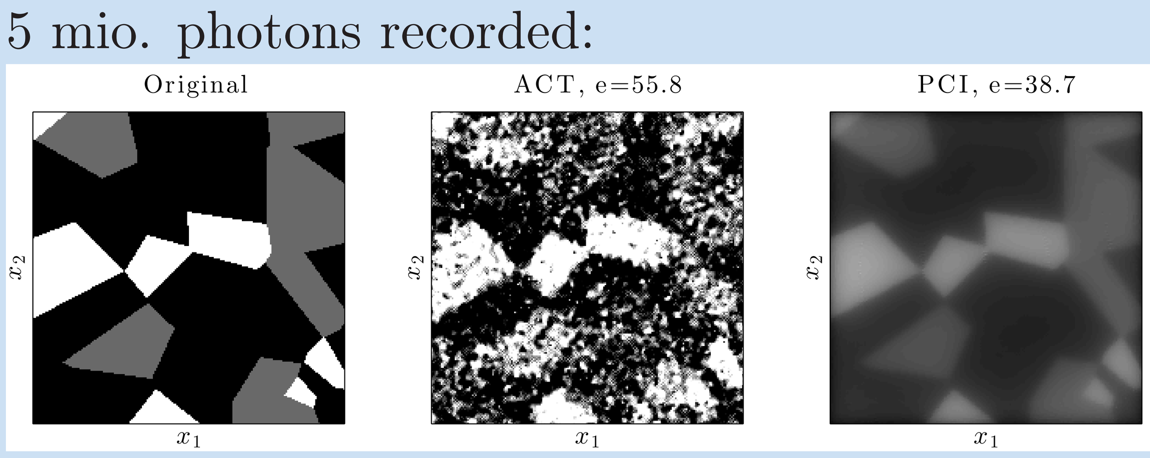
A comparison between ACT and CTF PCI. Noisy data complicates the reconstruction and may disturb results, especially for the materials with slight absorption differences. Ill-conditioned system magnifies noise in reconstruction solutions.

Regularization is typically used for noisy data, here TV(Total Variation)-regularization is used as a regularized reconstruction. A good prior for piecewise constant problems.

Error measure $e(x) = \frac{||x^* - x||}{||x^*||}$ is used to numerically verify solutions.

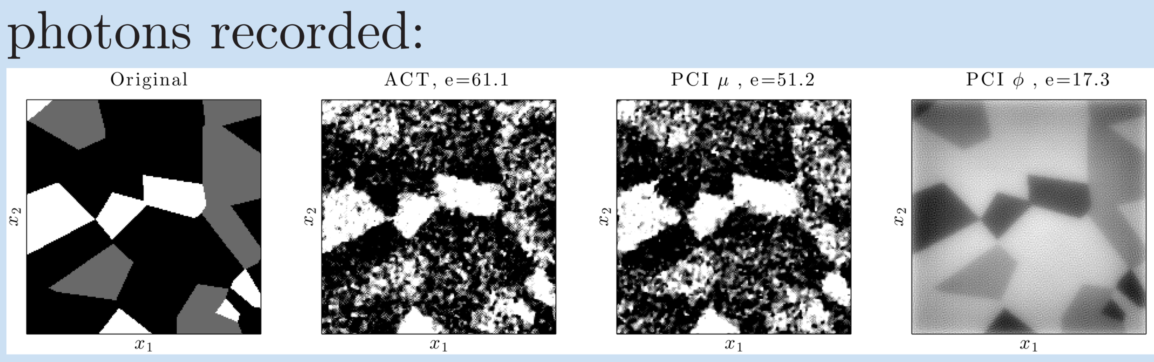
Example with two metals in a polycarbonate block. 40 keV X-ray source (assumed monochromatic), detector resolution $1\mu m$ and 200×200 pixels images. Materials are silicon, most absorbing, magnesium, slightly less absorbing, and polycarbonate, relatively low absorbing.

Duality method with one distance $R = 0.5m$ and 5 mio. photons recorded:



Clear distinction between between materials using PCI.

Normal CTF method with measurements for two distances $R_1 = 1mm$ and $R_2 = 0.5m$ and 5 mio. photons recorded:



Twice the data and results for the μ and ϕ. Similar disturbed results for μ but clearer distinction in phase results ϕ.

Algebraic Combined Model

Instead of the *two stage method*: 1) Phase retrieval followed by 2) reconstruction, a combined method where operators of phase retrieval, for example CTF, and reconstruction are combined to a single linear algebraic formulation $Ax = b$, which can then solved using for example TV-regularization. Further work on this hopefully will lead to even better PCI results.

Results

So far a forward model in physical units for modelling the travelling X-ray waves measured at variable distances has been implemented and tested, such that measurement data can be simulated. First PCI results has shown the advantage of using either one or two distances to obtain better results for materials which has very similar properties.

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