Exploratory Analysis of Climate Related Geodata

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Introduction

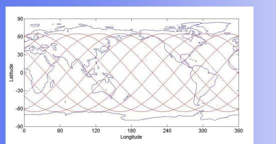
Is it possible to predict weather anomalies, climate changes or other related events by studying the ocean? How can these changes be detected and what impact does it have in a social economic relation?

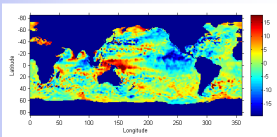
By a given set of observations over time, of the sea surface height anomalies, as well as sea surface temperatures, is it then possible to apply mathematical models to find patterns or predict an climate change ?

The data

Sea Surface Height (SSH)

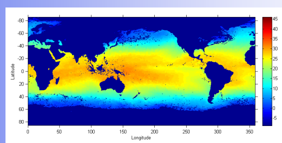
As the satellites rotate around the geocentre, they measure the sea surface height in a perpendicular line to the satellite lane. This gives the sea surface height nadir to the satellite. Combined with the rotation of the earth, data is obtained in an overlapping sinus pattern with a ~9.915 day interval. The unit is in meters.





Sea Surface Temperature (SST)

The Sea surface temperature for the same location is calculated using different models, taking measurement from various locations in to account, this including satellite, airplane, ships and fixed floating devices. The unit is in degree Celsius.



Canonical Correlation Analysis

Given two sets of multivariate variables, the parameters for mean and variance-covariance is, as follows:

$$X=\begin{pmatrix}X_1\\X_2\end{pmatrix},\mu=\begin{pmatrix}\mu_1\\\mu_2\end{pmatrix},\Sigma=\begin{pmatrix}\Sigma_{11}&\Sigma_{12}\\\Sigma_{21}&\Sigma_{22}\end{pmatrix}$$

We wish to maximize the correlation.

$$\text{Max } \rho=\frac{D\{a^TX_1,b^TX_2\}}{\sqrt{V\{a^TX_1\}V\{b^TX_2\}}}\Leftrightarrow \text{Max } \rho=\frac{a^T\Sigma_{12}b}{\sqrt{a^T\Sigma_{11}a\,b^T\Sigma_{22}b}}$$

Under the constrain that

$$a^T\Sigma_{11}a=b^T\Sigma_{22}b=1$$

By introducing Lagrange multipliers we get

$$L=a^T\Sigma_{12}b-\frac{\lambda_1}{2}\left(a^T\Sigma_{11}a-1\right)-\frac{\lambda_2}{2}\left(b^T\Sigma_{22}b-1\right)$$

Finding the partial derivative, solve the equation and re-write in to matrix form

$$\begin{aligned}\frac{\partial L}{\partial a}&=\Sigma_{12}b-\lambda_1\Sigma_{11}a\\\frac{\partial L}{\partial b}&=\Sigma_{21}a-\lambda_2\Sigma_{22}b\end{aligned}\Rightarrow\begin{bmatrix}0&\Sigma_{12}\\\Sigma_{21}&0\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=\rho\begin{bmatrix}\Sigma_{11}&0\\0&\Sigma_{22}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}$$

This is an eigen problem, but with rather large matrixes, by solving 'a' and 'b' and substituting we get

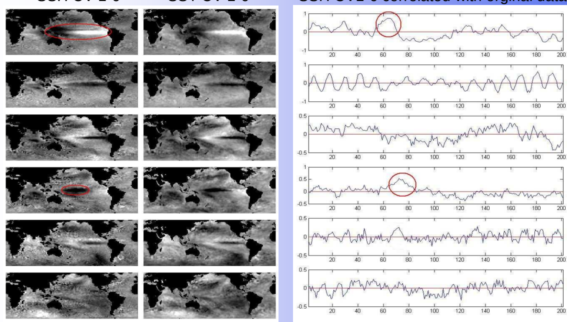
$$\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}a=\rho^2\Sigma_{11}a$$

$$\Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}b=\rho^2\Sigma_{22}b$$

This is now two smaller eigen problems (easier to solve) and by solving we get the two sets of eigen vectors that will maximize the correlation between the canonical variates in an descending order and still hold the constraint.

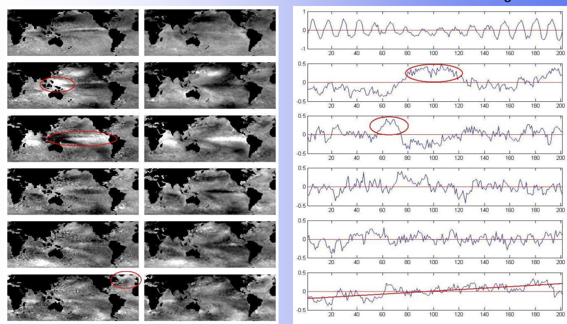
Results 1 – El Niño

SSH CV 1-6 | SST CV 1-6 | SSH CV1-6 correlated with orginal data



Results 2 – El Niño masked out

SSH CV 1-6 | SST CV 1-6 | SSH CV1-6 correlated with orginal data



Conclusion

In result 1, the El Niño and part of the La Niña is very clear in component one and four. In result 2, where most of the mid pacific ocean have been masked out and there by suppressing the El Niño, it still shows up, because its an world wide climate event. But other pattern now emerges, like the rising trend in the sea surface height around Greenland. This is do, to the melting of glacier in Greenland.

Common for both analysis is, that El Niño show up very strongly in the first components of the CCA. A spatial weighted analysis or a more supervised training, mite give a better result.