Generation of High-Quality Polygonal Meshes

Leif Kobbelt
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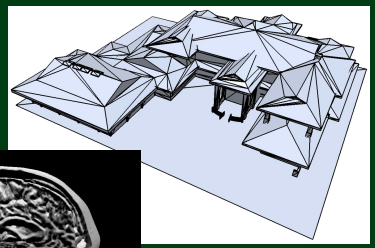
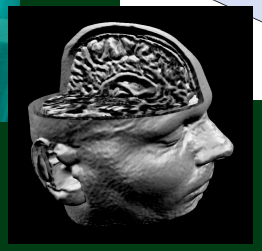
Geometry Processing Pipeline

- raw data → *shape information*
  (points, polygons, voxels)





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  (triangles)

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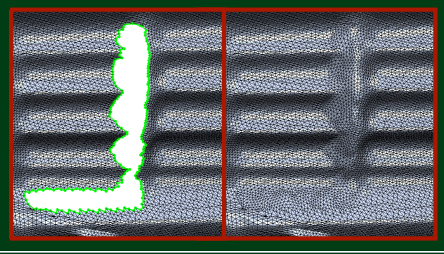
- mesh repair → *topological consistency*
  (manifolds)



Geometry Processing Pipeline

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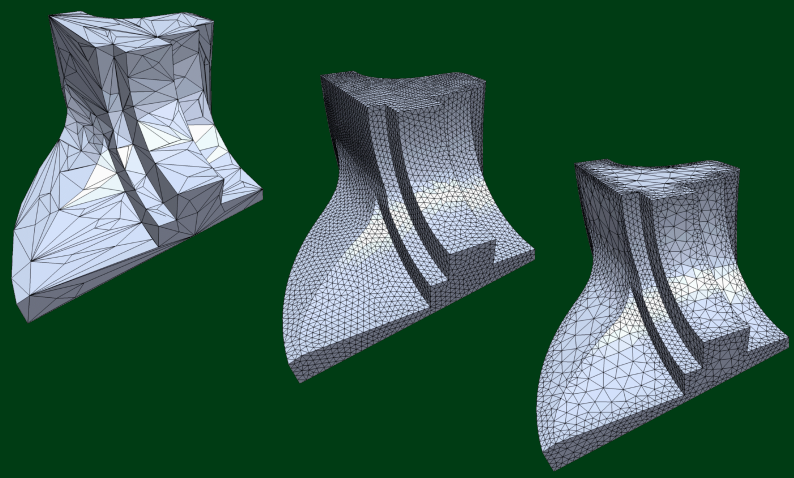
- mesh optimization → *geometric quality*
  (smoothing, decimation, remeshing)

Remeshing



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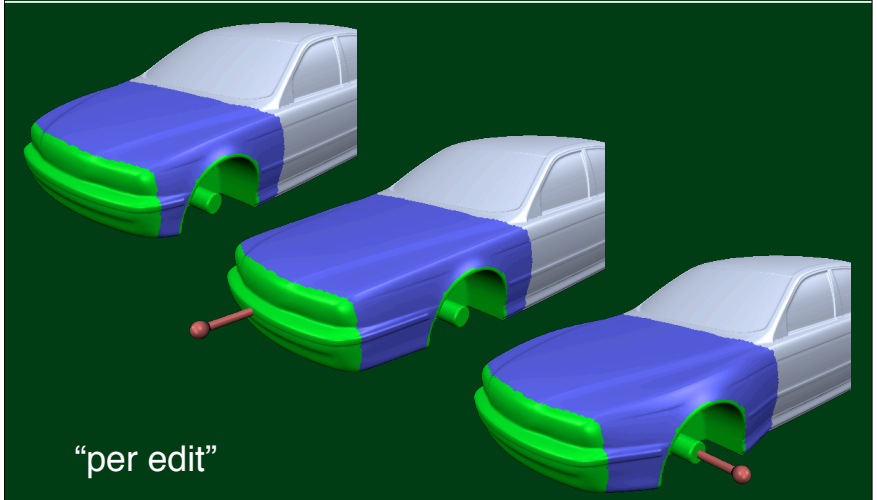
- mesh optimization → *geometric quality*
  (smoothing, decimation, remeshing)
- mesh editing → *intuitive handling / dynamics*
  (shape control handles)

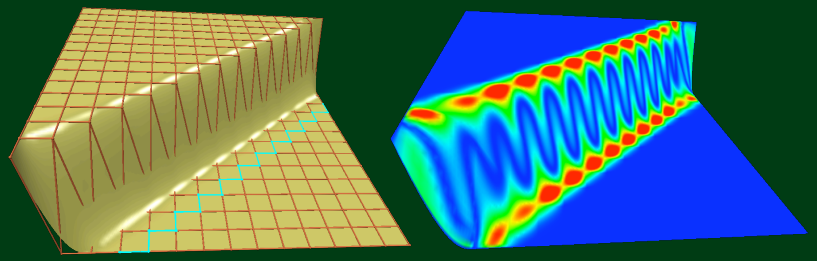
Shape Editing



“per edit”

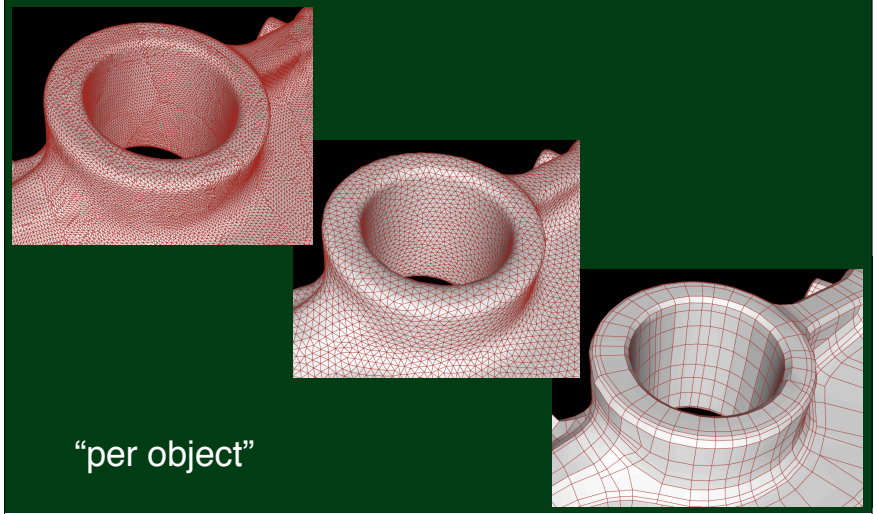
Shape Editing

$$\mathcal{S}(u,v) = \sum_{i,j} \mathbf{c}_{i,j} B_{i,j}(u,v)$$



"per object" ... *control vertices*

Shape Editing



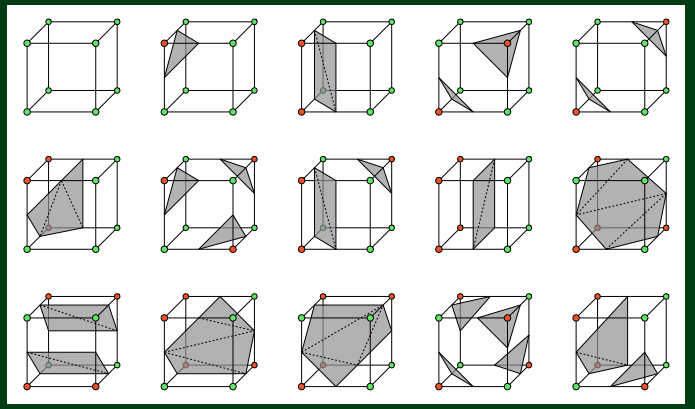
"per object"

Generate - Repair - Optimize

- ... from volume data
  - thresholding (marching cubes et al.)
  - deformable surfaces
- ... from point clouds
  - surface-based vs. volumetric
  - signed vs. unsigned distance function

Generate - Repair - Optimize

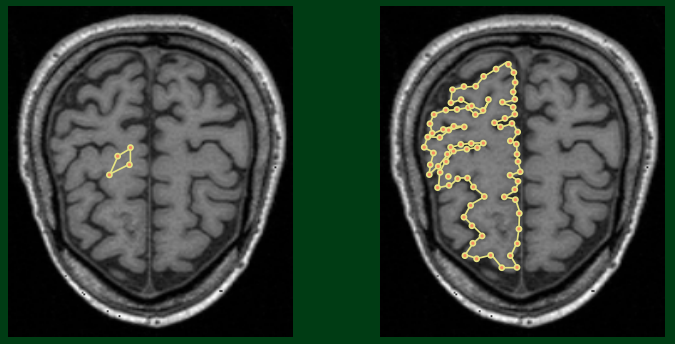
Marching Cubes



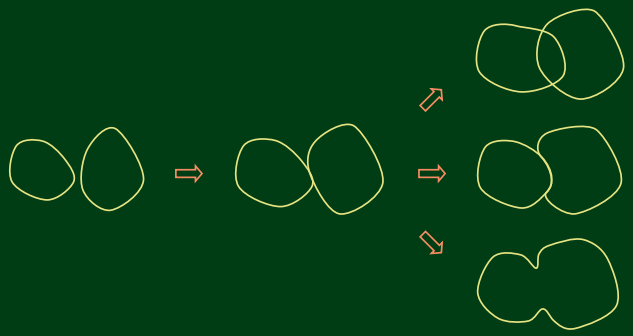
Generate - Repair - Optimize

- thresholding is sensitive to noise
- deformable surfaces preserve smoothness and connectedness
- **explicit formulation**: snakes
  - re-parameterization issues
- **implicit formulation**: level sets
  - topology control

Generate - Repair - Optimize



Generate - Repair - Optimize



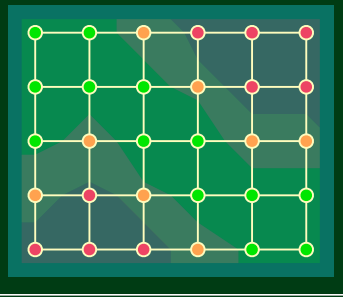
Generate - Repair - Optimize

Implicit Representation

- contour $\mathcal{C}(t) \subseteq R^3$
- arrival time $\eta(x, y, z) \in R$
- level set $\mathcal{C}(t) = \{\mathbf{p} \in R^3 : \eta(\mathbf{p}) = t\}$
- solve PDE for η

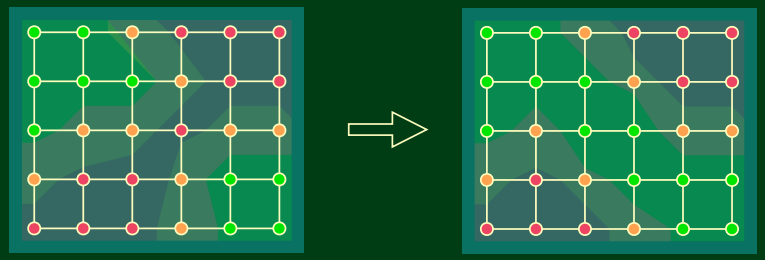
Fast Marching Method

- Each grid point is assigned one of three states.
  - conquered, fixed $\eta(\mathbf{p})$
  - front, tentative $\eta(\mathbf{p})$
  - far away, unknown $\eta(\mathbf{p}) = \infty$

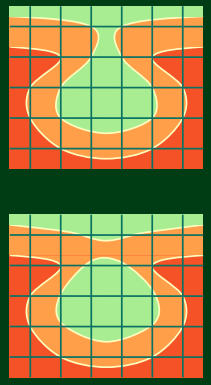


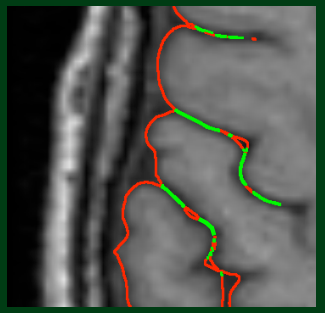
Fast Marching Method

- The fast marching method provides no topology control, i.e. the contour may merge.

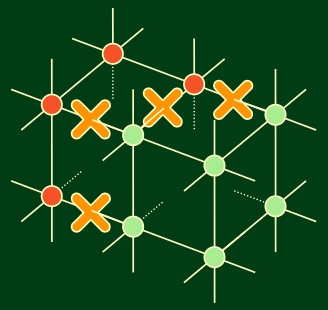


Generate - Repair - Optimize





Cut-Edge Grid



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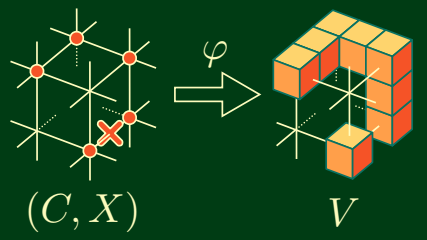
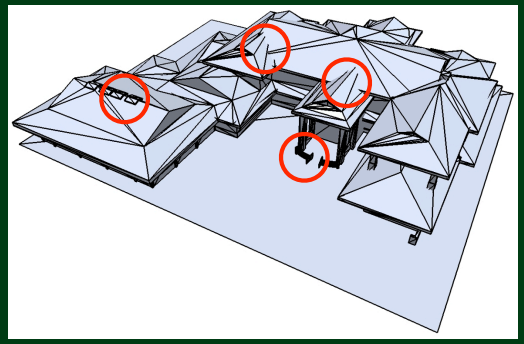


Generate - ***Repair*** - Optimize

- ... from unstructured triangle soups
- ... from tesselated NURBS models

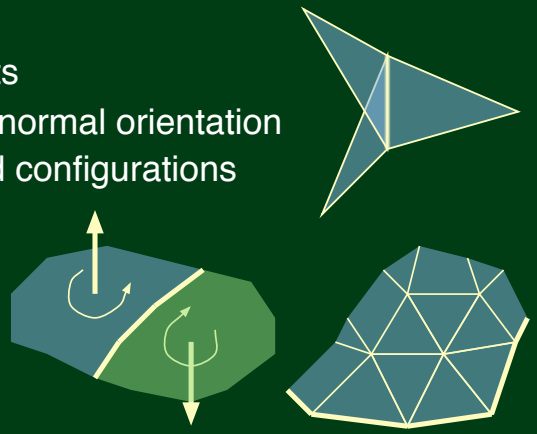
Generate - ***Repair*** - Optimize

- 3D models may look nice at the first glance ...



Generate - ***Repair*** - Optimize

- types of artifacts
  - inconsistent normal orientation
  - non-manifold configurations
  - boundaries
  - overlaps
  - gaps
  - intersections



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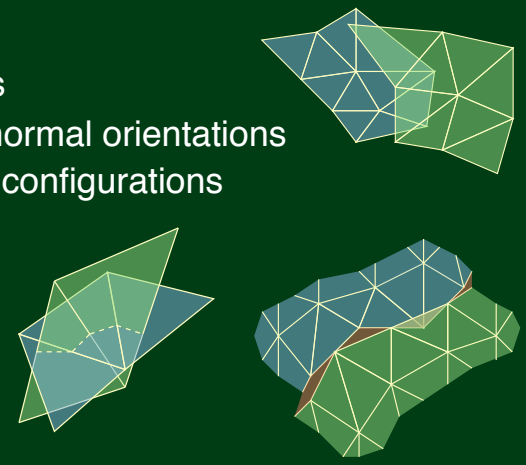
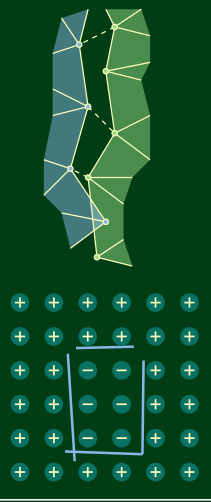


Generate - ***Repair*** - Optimize

- surface oriented approaches
  - structure preserving, minimal modification of the input
  - no guarantee on output quality
- volume oriented approaches
  - guaranteed manifold output
  - aliasing artifacts, limited resolution, global resampling



Generate - ***Repair*** - Optimize

- surface-based techniques
- volumetric techniques
- hybrid representations
  - voxel grid ... simple topology
  - triangle mesh ... best available geometry

Generate - ***Repair*** - Optimize

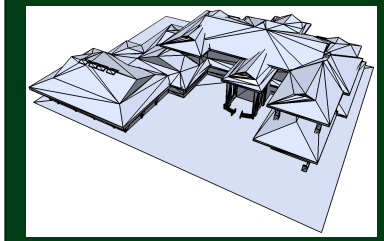
volumetric representation

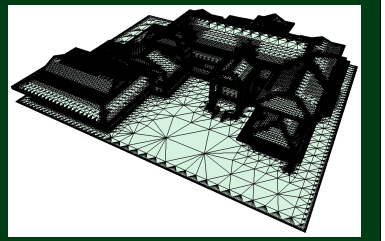
scan convert

fill gaps, remove interior

volumetric representation

extract mesh

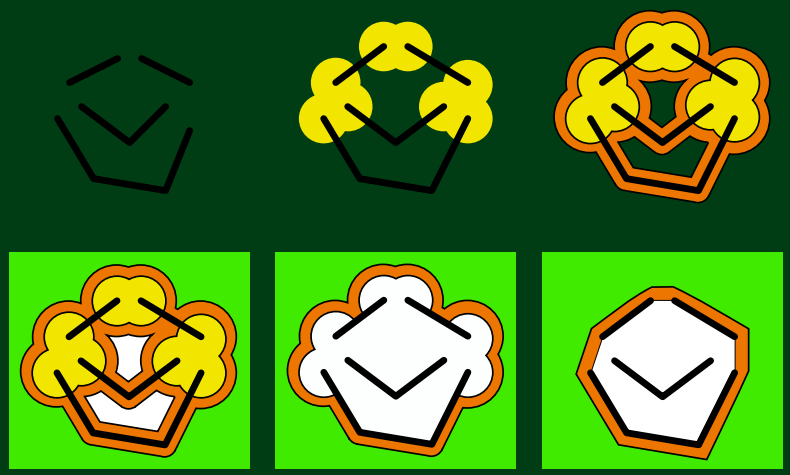




Generate - ***Repair*** - Optimize

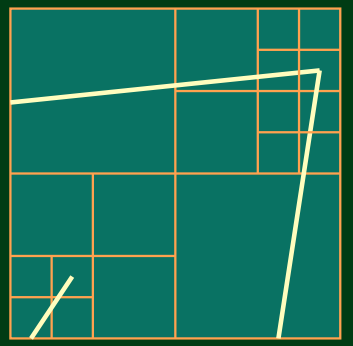
- given: input model M
  maximum approx. tolerance ϵ
  maximum hole/gap size ρ
- find: watertight, manifold model R with
  - distance(M,R) < ϵ
  - distance(R,M) < ρ
  - distance(R,M) > ϵ $\Rightarrow$ boundary of M
  - faithful normal reconstruction

Generate - ***Repair*** - Optimize



Generate - ***Repair*** - Optimize

- adaptive scan conversion



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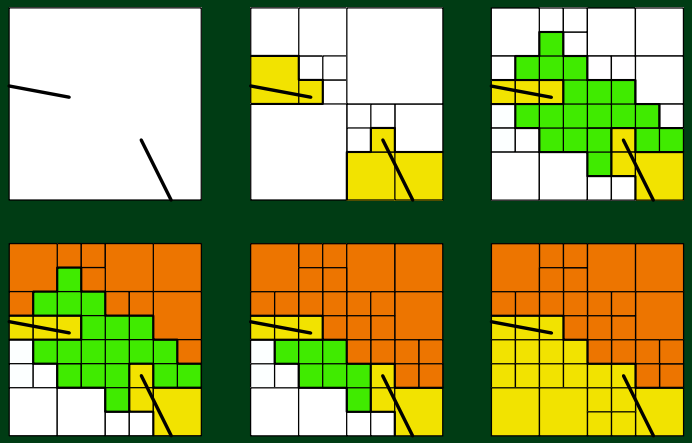

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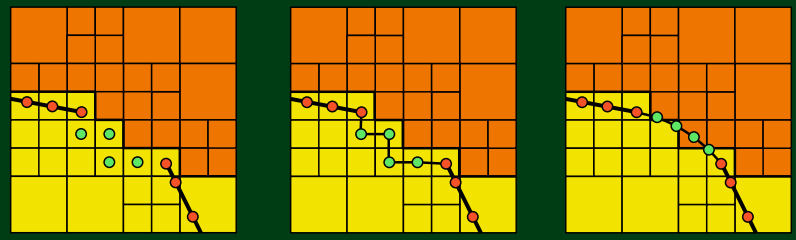

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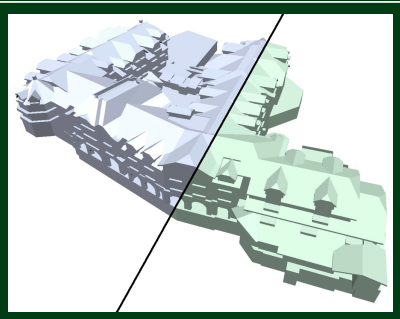
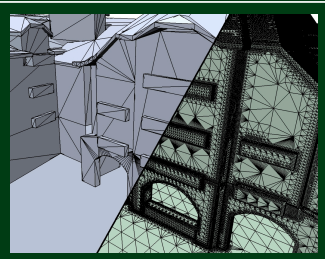
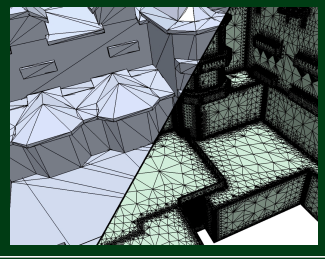
Generate - ***Repair*** - Optimize







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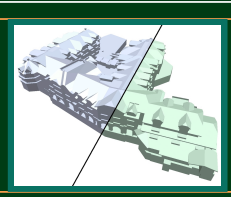

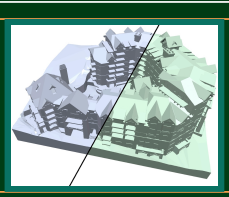

#input triangles	11904			50056		
resolution	500^3	1000^3	1500^3	500^3	1000^3	1500^3
#cells	1024K	4019K	7610K	4120K	17411K	29518K
#triangles	1187K	3882K	6890K	1459K	4780K	7421K
scan conversion	18s	255s	311s	110s	1545s	4852s
extraction	25s	91s	186s	43s	220s	331s
total	43s	346s	497s	153s	1765s	5183s

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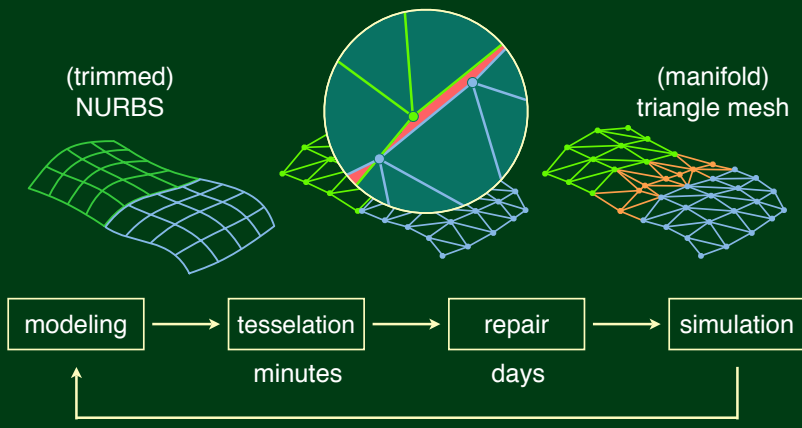
Generate - ***Repair*** - Optimize

original
3346 triangles

reconstruction
1370802 triangles
(at 1000^3)

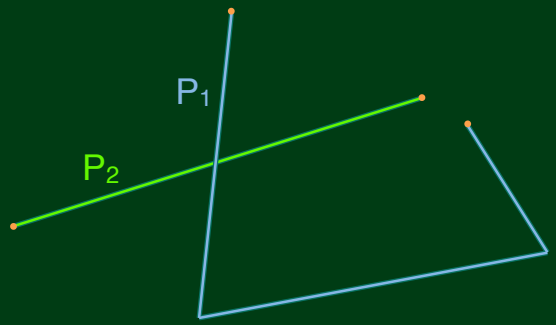
decimated
18032 triangles

Generate - ***Repair*** - Optimize



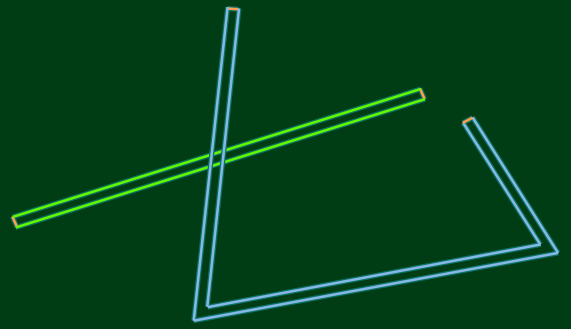
Generate - ***Repair*** - Optimize

- input: set of patches $P_1,...,P_n$



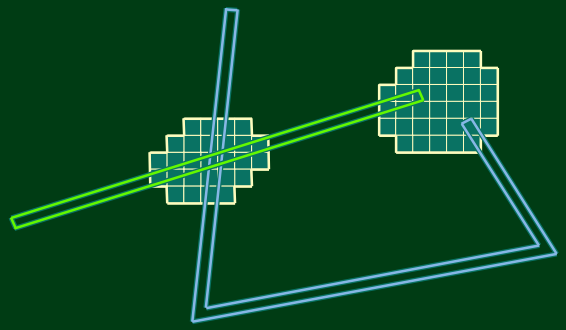
Generate - ***Repair*** - Optimize

- remove boundaries by duplicating each patch and stitching them along their common boundary.



Generate - ***Repair*** - Optimize

- setup a ϵ-grid within the critical regions



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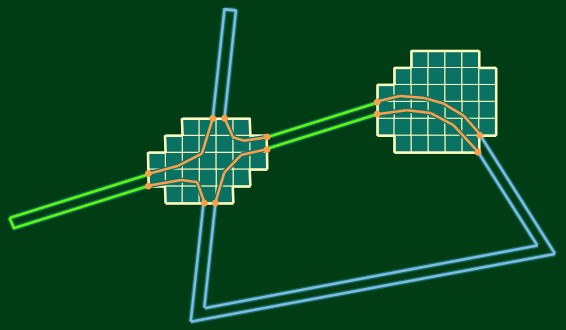
Generate - ***Repair*** - Optimize

- reconstruct surface within the critical regions and merge it with the outside



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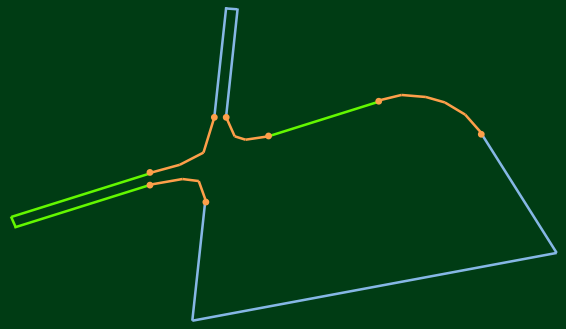
Generate - ***Repair*** - Optimize

- remove internal geometry
- decimation / optimization



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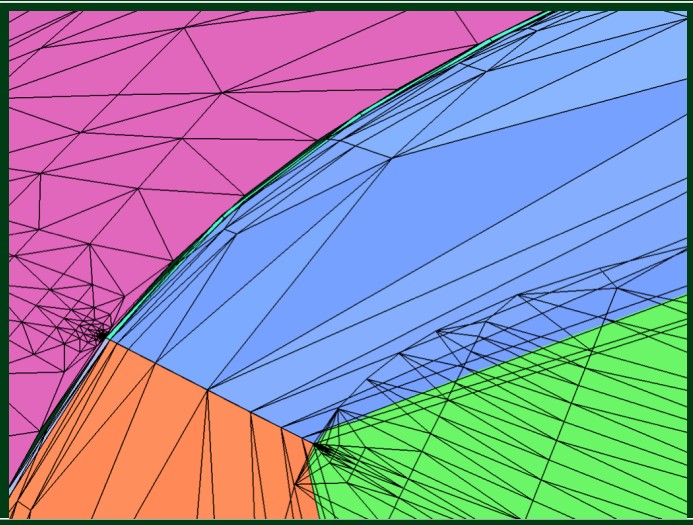

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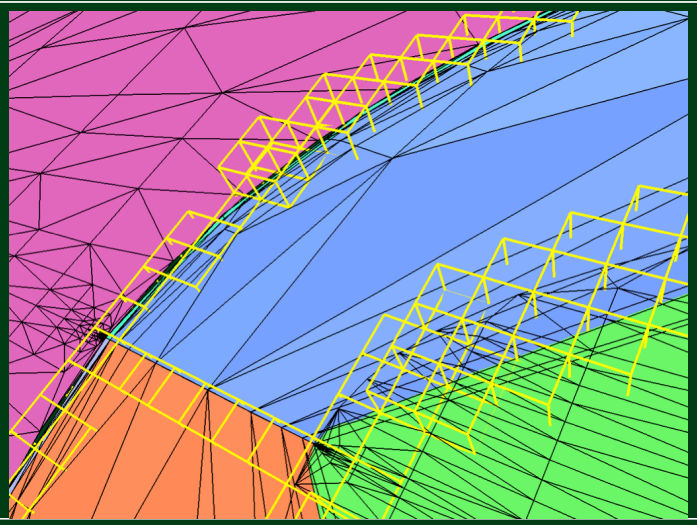
Generate - ***Repair*** - Optimize



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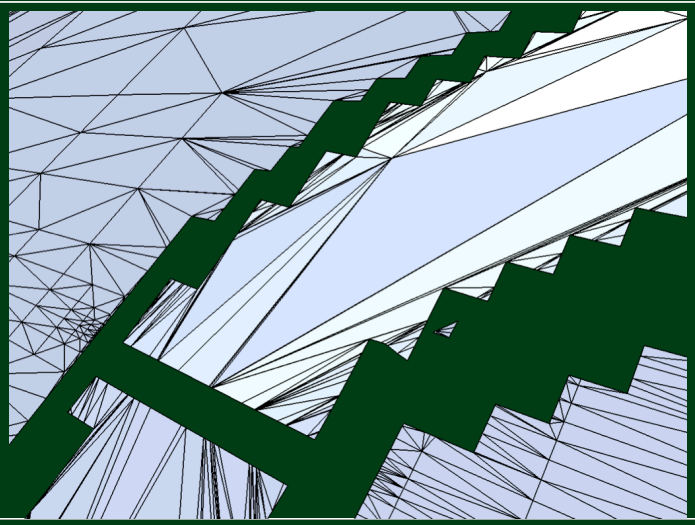

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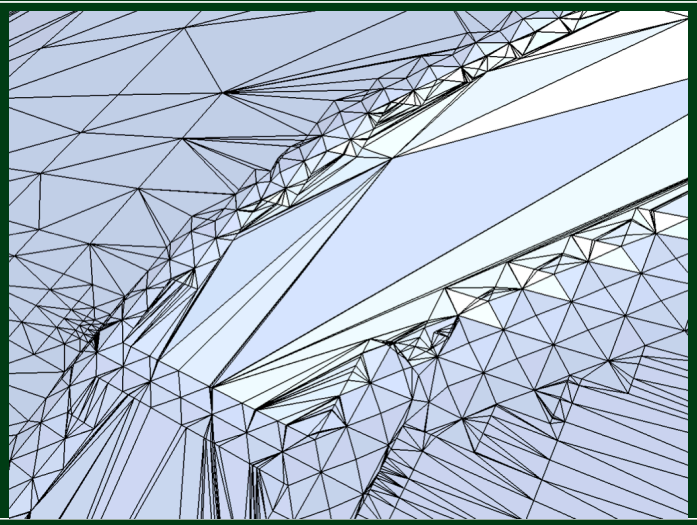
Generate - ***Repair*** - Optimize



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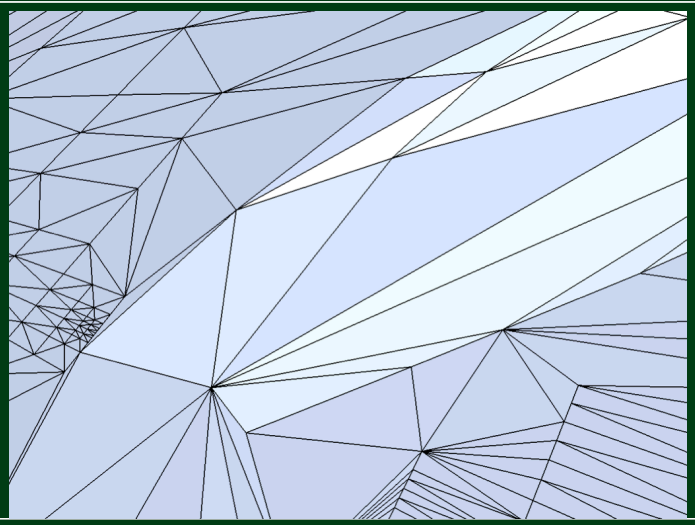

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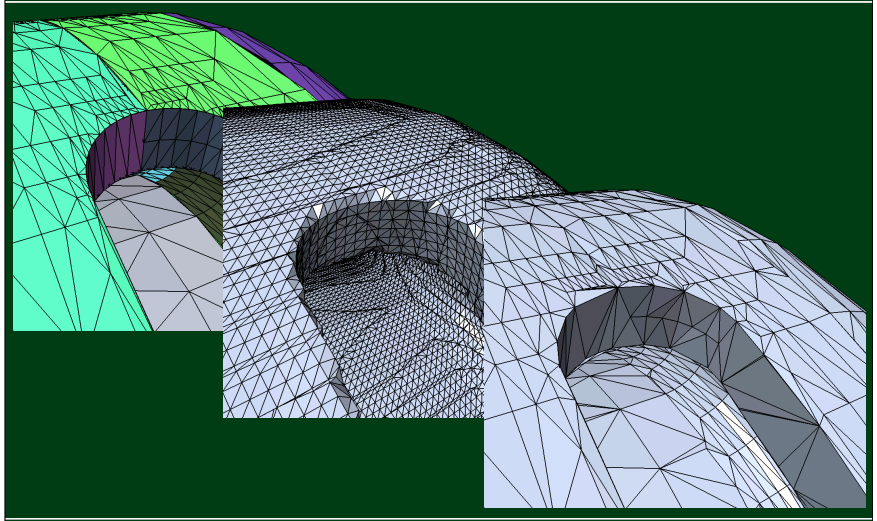
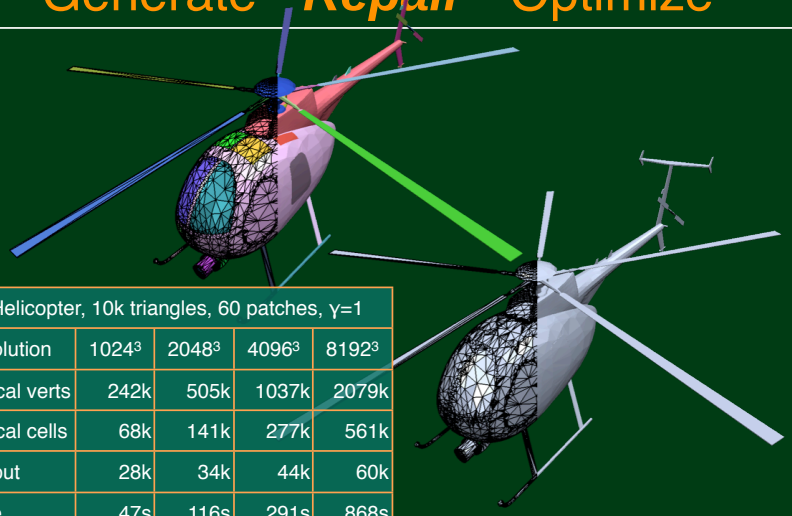
Generate - ***Repair*** - Optimize

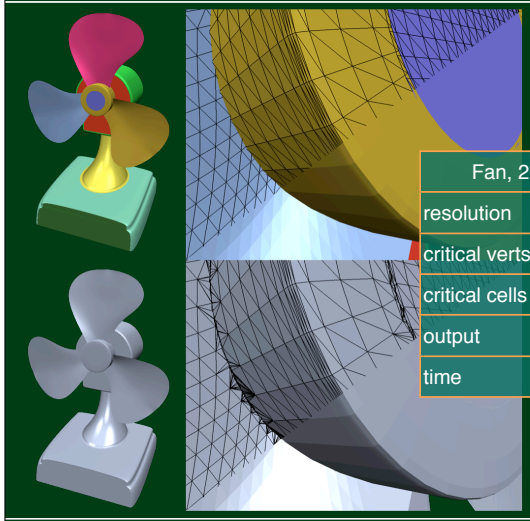


Generate - ***Repair*** - Optimize



Helicopter, 10k triangles, 60 patches, γ=1				
resolution	1024^3	2048^3	4096^3	8192^3
critical verts	242k	505k	1037k	2079k
critical cells	68k	141k	277k	561k
output	28k	34k	44k	60k
time	47s	116s	291s	868s

Generate - ***Repair*** - Optimize



Fan, 269k triangles, 12 patches, γ=2				
resolution	1024^3	2048^3	4096^3	8192^3
critical verts	238k	460k	828k	1649k
critical cells	64k	113k	229k	523k
output	503k	512k	529k	556k
time	83s	123s	193s	303s

Generate - Repair - ***Optimize***

- isotropic remeshing
- anisotropic remeshing

Generate - Repair - ***Optimize***

- isotropic remeshing prefers ...
  - equal edge length
    - remove too short edges — edge collapse
    - remove too long edges — 2-4 edge split
  - regular valences
    - valence balance — edge flip
  - uniform vertex distribution
    - tangential smoothing — Laplace operator

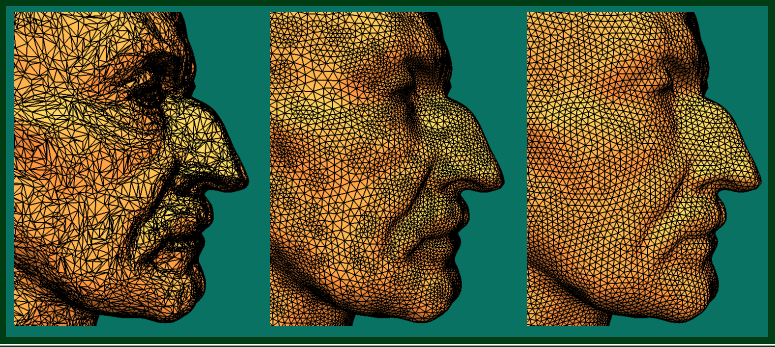
Generate - Repair - ***Optimize***

0. specify target edge length L

1. split all edges long than L_{max}
2. collapse all edges shorter than L_{min}
3. flip edges to promote valence 6
4. relax vertex positions by tangential smoothing
5. goto 1

Generate - Repair - ***Optimize***

- optimal thresholds !?
  - (L_{min}, L_{max}) = (0.5, 2.0)
  - (L_{min}, L_{max}) = (4/5, 4/3)



Generate - Repair - ***Optimize***

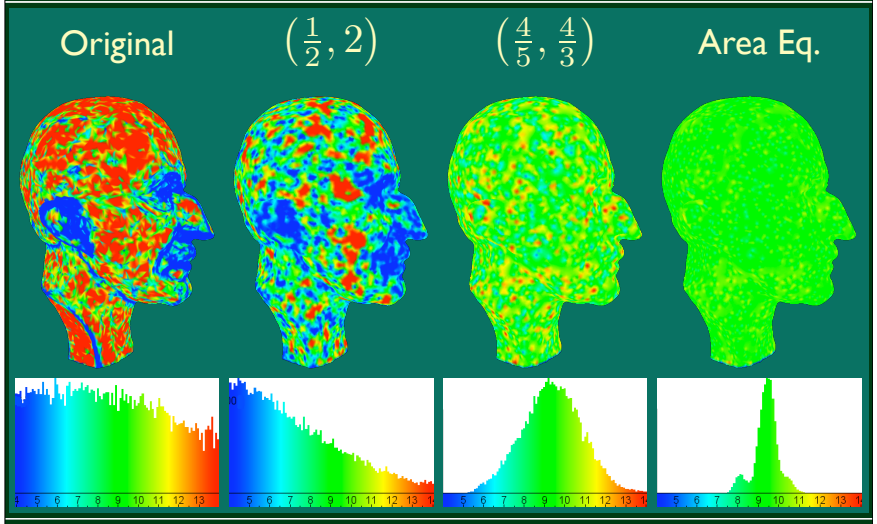
- tangential smoothing with area equalization (leads to symmetric Laplace matrix)
- area-weighted centroid

$$\mathbf{g}_i := \frac{1}{\sum_{\mathbf{q}_i} A(\mathbf{q}_i)} \sum_{\mathbf{q}_i} A(\mathbf{q}_i)\, \mathbf{q}_i$$

- tangential update

$$\mathbf{p}_i \mapsto \mathbf{p}_i + \lambda \left(I - \mathbf{n}_i \mathbf{n}_i^T \right) (\mathbf{g}_i - \mathbf{p}_i)$$

Generate - Repair - ***Optimize***

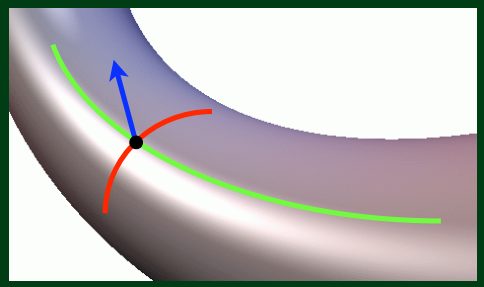


Generate - Repair - ***Optimize***

- an-isotropic remeshing prefers ...
  - quad faces
  - curvature dependent size and aspect ratio (approximation measure)
  - local orientation (curvature directions, shape operator)
  - global alignment (feature detection and handling)

Generate - Repair - ***Optimize***

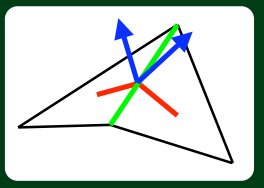
- local orientation
- 2nd fundamental form defines a local **orthogonal** frame (min-/max-curvature directions plus normal)



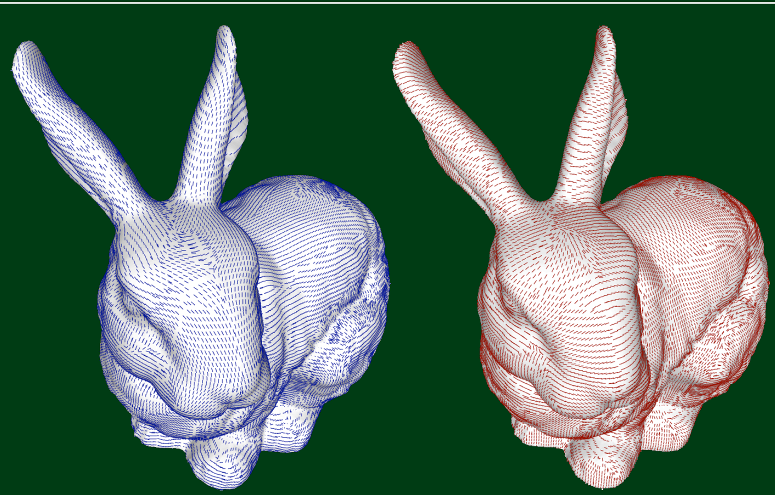
Generate - Repair - ***Optimize***

- projection to edges $\mathbf{e}\,\mathbf{e}^T \qquad \|\mathbf{e}\| = 1$ (minimum curvature direction)
- weighted sum of edge projection operators

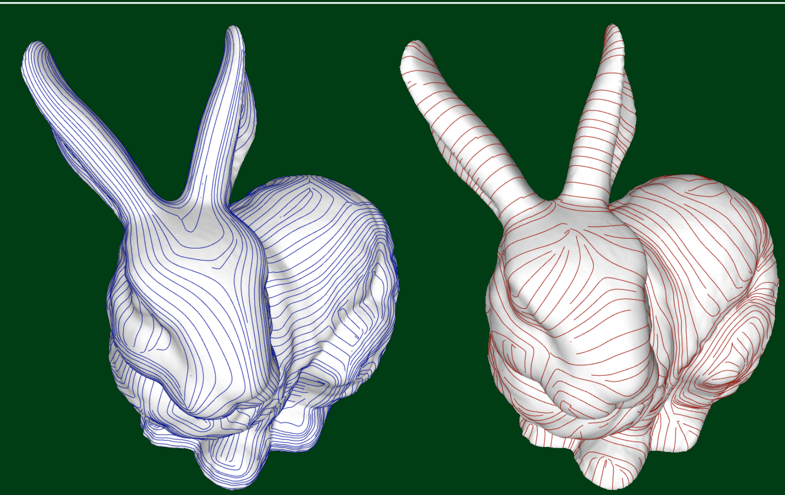
$$\mathcal{S}(\mathbf{p}) = \sum_{\mathbf{e} \in B(\mathbf{p})} \beta(\mathbf{e}) \, \|\mathbf{e} \cap B(\mathbf{p})\| \, \mathbf{e}\,\mathbf{e}^T$$



Generate - Repair - *Optimize*



Generate - Repair - *Optimize*



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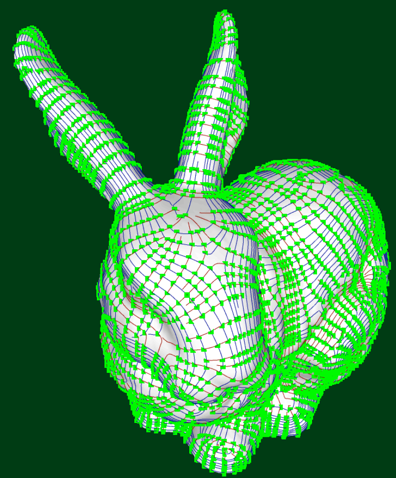
Generate - Repair - *Optimize*



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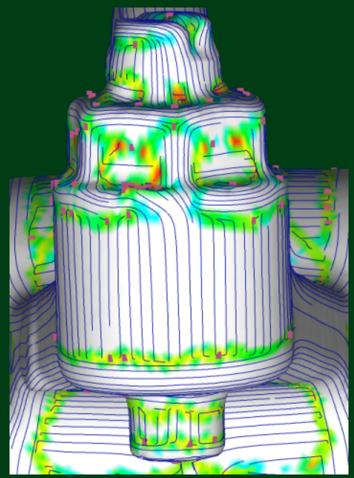
Generate - Repair - *Optimize*



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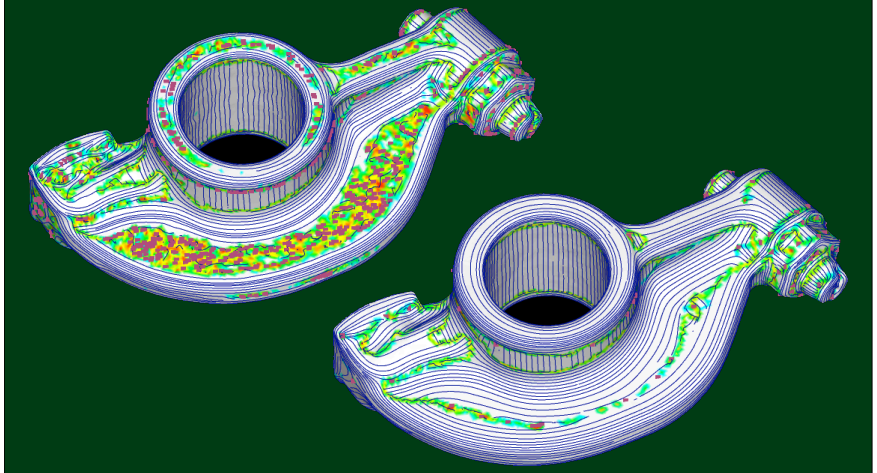
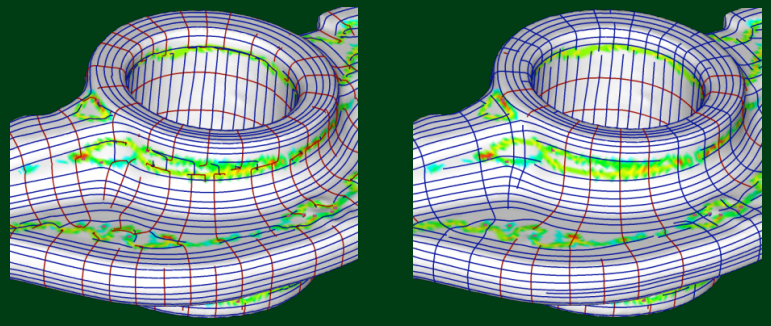
Generate - Repair - ***Optimize***

- compute curvature direction field
- estimate local reliability
- propagate orientation information from anisotropic regions to isotropic ones
- trace curve network along minimum and maximum curvature directions (starting from anisotropic regions)

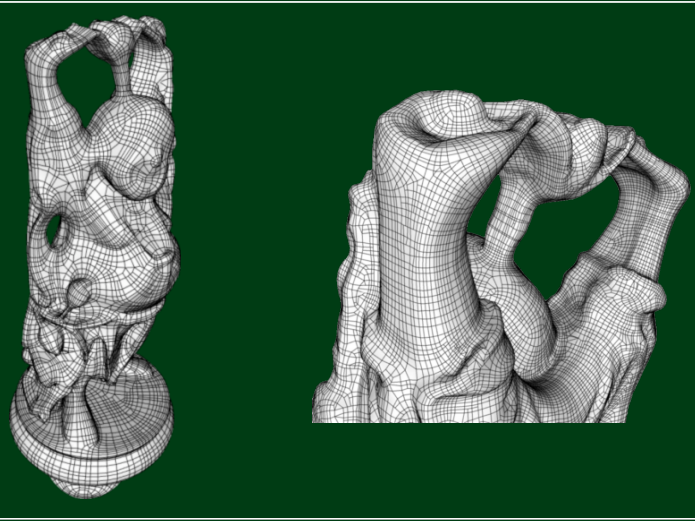
Generate - Repair - ***Optimize***



Generate - Repair - ***Optimize***



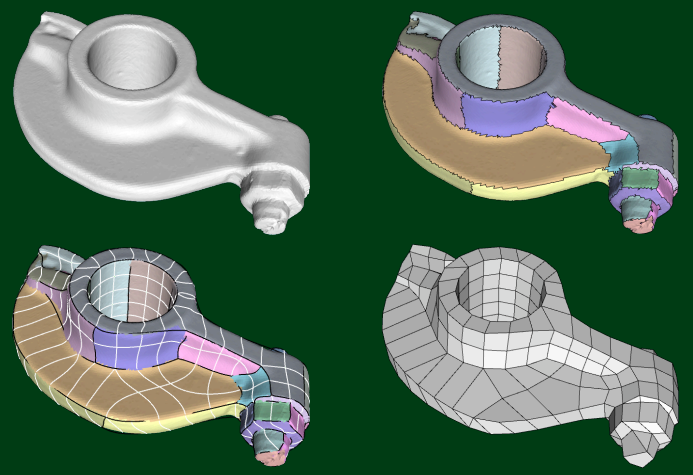
Generate - Repair - ***Optimize***



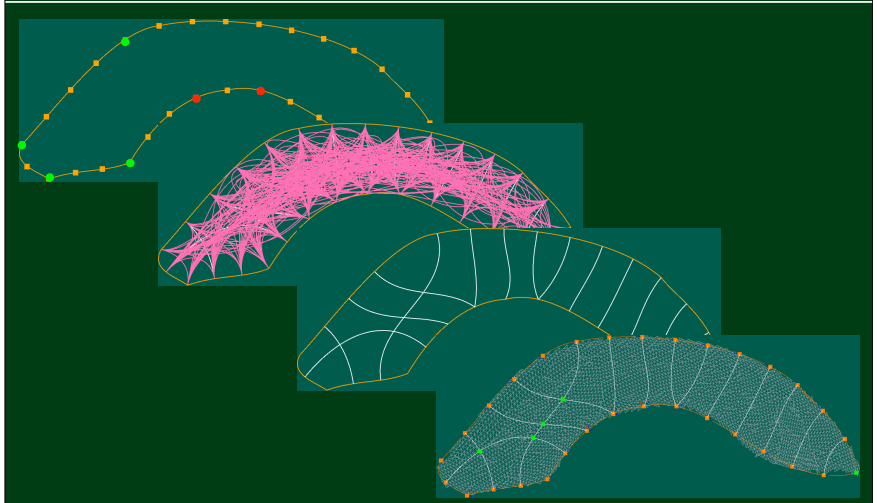
Generate - Repair - ***Optimize***

- marching techniques cannot capture the global structure of the model
- critical for *coarse* quad meshes
- two-step procedure:
  - segmentation (global structure)
  - quad meshing per segment (local shape and alignment)

Generate - Repair - ***Optimize***

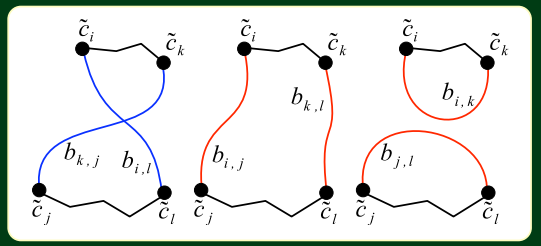


Generate - Repair - ***Optimize***

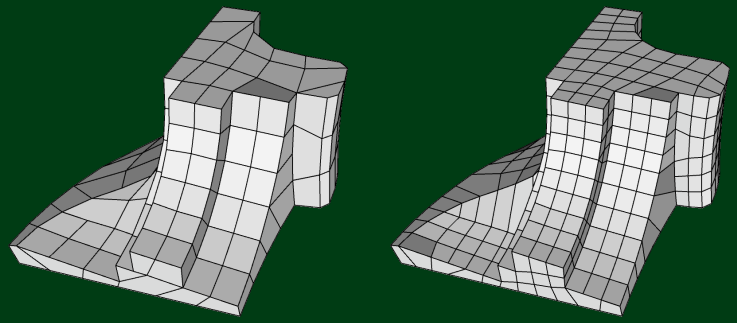


Generate - Repair - ***Optimize***

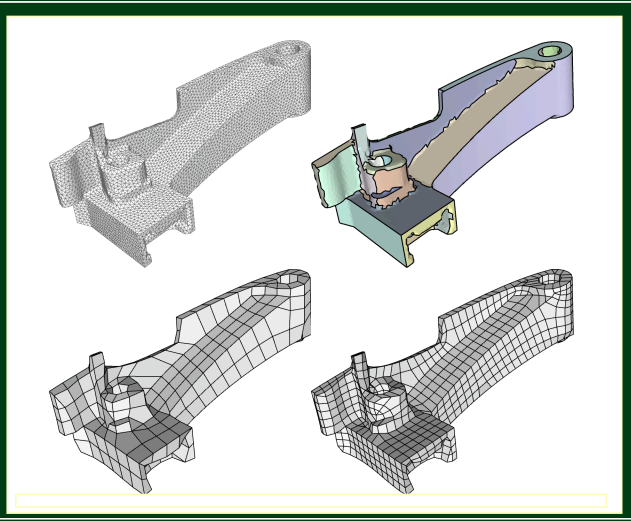
- combinatorial optimization
- energy functional
  - orthogonality at intersections
  - parallelism within faces



Generate - Repair - ***Optimize***



Generate - Repair - ***Optimize***



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