Estimation of complex motions in video signals

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Nonlinear analysis of multidimensional signals:

LOcal adaptive estimation of COmplex MOTion and ORientation patterns

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Im DFG Schwerpunkt 1114: "Mathematical methods for time series analysis and digital image processing"

http://www.math.uni-bremen.de/zetem/DFG-Schwerpunkt/

Project Ba-1176/7

Intrinsic dimension in 3D

i0D: constant in all directions: $f(x,y,t) = const.$

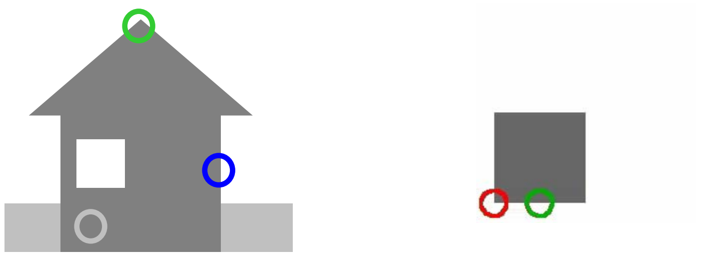
i1D: constant in 2 directions: $f(x,y,t) = g(\xi)$

i2D: constant in one direction: $f(x,y,t) = g(\xi,\zeta)$

i3D: no constant direction: $f(x,y,t) = g(\xi,\zeta,\tau)$

Intrinsic dimension

i2D and *i3D* regions are the least frequent but most significant.



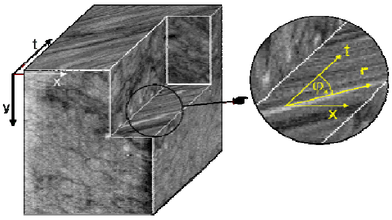
i0D *i1D* *i2D* *i3D*

The classical motion model

f(x,y,t) image-sequence intensity

$\mathbf{v} = (v_1, v_2)^T$ motion vector

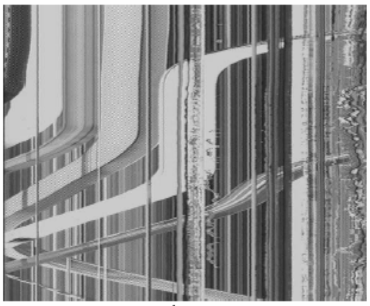
$$\alpha(v)f = v_1 \frac{\partial}{\partial x} f + v_2 \frac{\partial}{\partial y} f + \frac{\partial}{\partial t} f = V \cdot \nabla f = 0$$



$\mathbf{r} = (v_1, v_2, 1)$

Why complex motions?

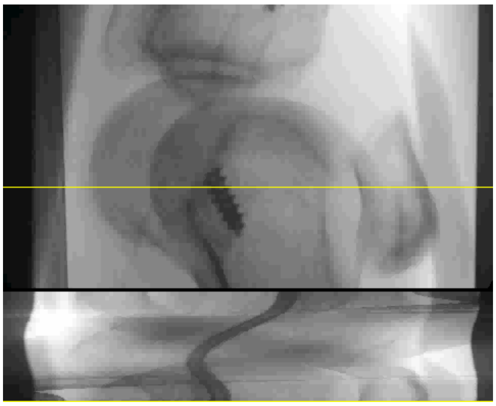
xy image



xt image

Basic problems with natural image sequences: multiple motions, occlusions, noisy data, features at different scales, ...

Transparent motions



Movie (top) and xt plot (below).

Motion model for transparent motions

Optical flow for one motion

f(x,y,t) image sequence, $\mathbf{v} = (v_1,v_2)^T$ motion vector

$$\alpha(\mathrm{v})\mathrm{f} = v_1 \frac{\partial}{\partial \mathrm{x}}\mathrm{f} + v_2 \frac{\partial}{\partial \mathrm{y}}\mathrm{f} + \frac{\partial}{\partial \mathrm{t}}\mathrm{f} = \mathrm{V} \cdot \nabla \mathrm{f} = 0$$

with the derivative operator: $\alpha(\mathrm{v}) = v_1 \frac{\partial}{\partial \mathrm{x}} + v_2 \frac{\partial}{\partial \mathrm{y}} + \frac{\partial}{\partial \mathrm{t}}$

Optical flow for N motions: $\mathrm{f} = g_1(x-\mathbf{v}_1 t)+\ldots+g_N(x-\mathbf{v}_N t)$

$$\boxed{\alpha(\mathrm{v}_1)\alpha(\mathrm{v}_2)\ldots\alpha(\mathrm{v}_N)\mathrm{f} = 0}$$

Mixed-motion parameters

Example for two motions **u, v** : $f(\mathbf{x},t)=g_1(\mathbf{x}-\mathbf{v}\,t)+g_2(\mathbf{x}-\mathbf{u}\,t)$

$$\begin{aligned}\alpha(\mathrm{v})\alpha(\mathrm{u})\,f &= u_1 v_1 f_{xx} + u_2 v_2 f_{yy} \\ &\quad + (u_1 v_2 + u_2 v_1) f_{xy} \\ &\quad + (u_1 + v_1) f_{xt} + (u_2 + v_2) f_{yt} + f_{tt}\end{aligned}$$

We define the *mixed-motion parameters* as:

$$\mathrm{c}_{\mathrm{xx}} = v_1 u_1 \qquad \mathrm{c}_{\mathrm{yy}} = v_2 u_2 \qquad \mathrm{c}_{\mathrm{xy}} = u_1 v_2 + u_2 v_1$$
$$\mathrm{c}_{\mathrm{xt}} = u_1 + v_1 \qquad \mathrm{c}_{\mathrm{yt}} = u_2 + v_2 \qquad \mathrm{c}_{\mathrm{tt}} = 1$$

Solving for the mixed-motion parameters

With the *mixed-motion parameters* we obtain:

$$\boxed{\begin{aligned}\alpha(\mathrm{v})\alpha(\mathrm{u})\mathrm{f} &= \mathrm{c}_{\mathrm{xx}}\mathrm{f}_{\mathrm{xx}} + \mathrm{c}_{\mathrm{yy}}\mathrm{f}_{\mathrm{yy}} + \mathrm{c}_{\mathrm{xy}}\mathrm{f}_{\mathrm{xy}} \\ &\quad + \mathrm{c}_{\mathrm{xt}}\mathrm{f}_{\mathrm{xt}} + \mathrm{c}_{\mathrm{yt}}\mathrm{f}_{\mathrm{yt}} + \mathrm{c}_{\mathrm{tt}}\mathrm{f}_{\mathrm{tt}} \\ &= \mathrm{V}\cdot\mathrm{L} = 0\end{aligned}}$$

For one motion we had

$$\mathrm{V}\cdot\nabla\mathrm{f} = 0$$

The above constraint can be used in a number of ways to derive the mixed motion parameters in V from f, e.g. by defining the generalized structure tensor $\boxed{\mathrm{J}_{\mathrm{N}}\mathrm{V} = 0}$ and solving the system $\boxed{\mathrm{J}_{\mathrm{N}} = \mathrm{h} * \mathrm{L}^{\mathrm{T}}\mathrm{L}}$

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e.g. $\mathrm{V}_{\mathrm{i}} \propto (\mathrm{M}_{\mathrm{im}}, -\mathrm{M}_{\mathrm{i(m-1)}}, \ldots, (-1)^{\mathrm{m}}\mathrm{M}_{\mathrm{i1}})$

$M_{ij},\ i = 1,\ldots,m$ are the minors of $\mathbf{J}_{\mathrm{N}}$

Separation of the mixed-motion parameters

We interpret the motion vectors **u** and **v** as *complex numbers* ($\mathbf{v} = v_1 + i\, v_2$, $\mathbf{u} = u_1 + i\, u_2$) and observe that

$$\mathrm{u\,v} = \mathrm{A}_0 = \mathrm{c}_{xx} - \mathrm{c}_{yy} + i\mathrm{c}_{xy}$$

$$\mathrm{u} + \mathrm{v} = \mathrm{A}_1 = \mathrm{c}_{xt} + i\mathrm{c}_{yt}$$

A_0 and A_1 are homogeneous symmetrical functions of the coordinates **u** and **v** and therefore by Vieta's rule the coefficient of the complex polynomial

$$\boxed{\mathrm{Q(z) = (z-u)(z-v) = z^2 - A_1 z + A_0}}$$

that has the complex roots **u** and **v**.

In case of N motions

$$\boxed{\mathrm{Q(z) = z^N - A_{N-1} z^{N-1} + \cdots + (-1)^N A_0}}$$

Fourier analysis of multiple transparent motions

Continuous time

$$\mathbf{f}(\mathbf{x}, t) = \mathbf{g}_1(\mathbf{x} - t\mathbf{u}) + \mathbf{g}_2(\mathbf{x} - t\mathbf{v})$$

$$\Leftrightarrow \alpha(\mathbf{u})\alpha(\mathbf{v})\mathbf{f} = 0$$

$$\Leftrightarrow \mathbf{F} = G_1 \delta(\mathbf{u} \cdot \omega + \omega_t) + G_2 \delta(\mathbf{v} \cdot \omega + \omega_t)$$

Discrete time

$$\mathbf{f}_k(\mathbf{x}) = \mathbf{g}_1(\mathbf{x} - k\Delta t\mathbf{u}) + \mathbf{g}_2(\mathbf{x} - k\Delta t\mathbf{v})$$

$$\Leftrightarrow \mathbf{F}_k(\omega) = \phi_1^k G_1(\omega) + \phi_2^k G_2(\omega)$$

$$\alpha(\mathbf{u}) = \mathbf{u} \cdot \nabla + \frac{\partial}{\partial t} \qquad \phi_1 = e^{-j\mathbf{u}\cdot\omega}$$

Block-matching for transparent motions

$$\mathbf{F}_k(\omega) = \phi_1^k G_1(\omega) + \phi_2^k G_2(\omega)$$

Layer elimination

$$\mathbf{F}_2(\omega) - a_1\mathbf{F}_1(\omega) + a_2\mathbf{F}_0(\omega) = \mathbf{0}$$

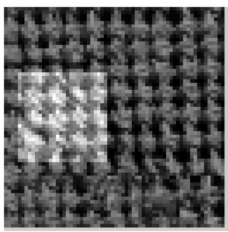
$$a_1 = \phi_1 + \phi_2 \quad a_2 = \phi_1\phi_2$$

Block-matching constraint

Back to space domain

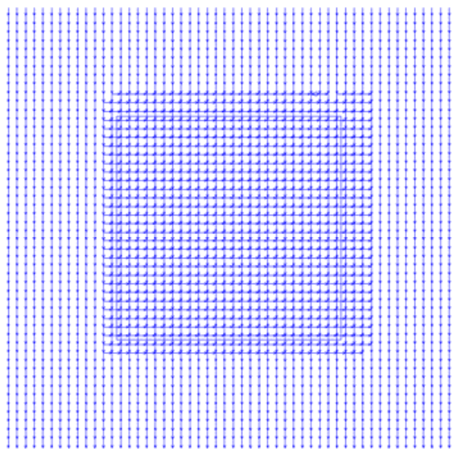
$$\mathbf{f}_2(\mathbf{x}) - \mathbf{f}_1(\mathbf{x}-\mathbf{u}) - \mathbf{f}_1(\mathbf{x}-\mathbf{v}) + \mathbf{f}_0(\mathbf{x}-\mathbf{u}-\mathbf{v}) = \mathbf{0}$$

Synthetic results for block-matching



SNR : 35 dB

Block size : 5 x 5 pixels



Fourier analysis of multiple occluded motions

$$\mathbf{f}(\mathbf{x},t) = \chi(\mathbf{x}-t\mathbf{u})\mathbf{g}_1(\mathbf{x}-t\mathbf{u}) + \overline{\chi}(\mathbf{x}-t\mathbf{u})\mathbf{g}_2(\mathbf{x}-t\mathbf{v})$$

$\mathbf{g}_1(\mathbf{x})$ occluding object

$\mathbf{g}_2(\mathbf{x})$ occluded object

χ characteristic function of the occluding object

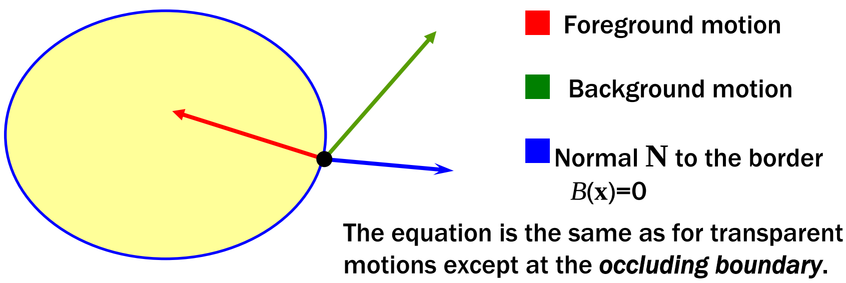
$\overline{\chi} = 1 - \chi$

How does the Fourier transform of $\mathbf{f}(\mathbf{x},t)$ deviate from the two motion planes?

$$\alpha(\mathbf{u})\alpha(\mathbf{v})\mathbf{f}(\mathbf{x},t) \ ?$$

Occluded motion

$$\alpha(\mathbf{u})\alpha(\mathbf{v})\mathbf{f}(\mathbf{x},t) = q(\mathbf{x},\mathbf{u},\mathbf{v},\mathrm{t})\delta(B(\mathbf{x}-t\mathbf{u}))$$
$$q(\mathbf{x},\mathbf{u},\mathbf{v},\mathrm{t}) = -(\mathbf{u}-\mathbf{v})\cdot\mathbf{N}(\mathbf{x}-t\mathbf{u})\,(\mathbf{u}-\mathbf{v})\cdot\nabla\mathbf{g}_2(\mathbf{x}-t\mathbf{v})$$



The equation is the same as for transparent motions except at the ***occluding boundary***.

$q(\mathbf{x},\mathbf{u},\mathbf{v},\mathrm{t})$ is maximal if $\mathbf{u}$ - $\mathbf{v}$ is orthogonal to the boundary

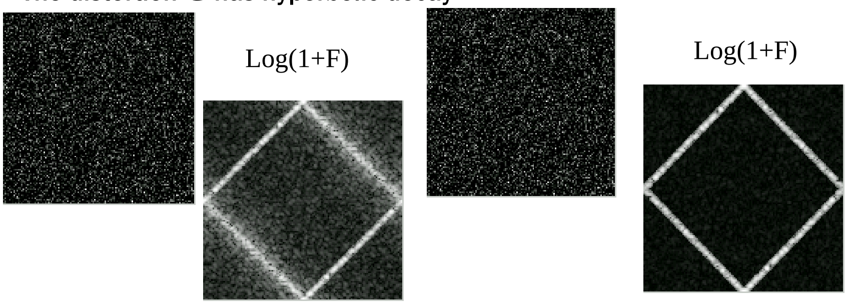
is null if $\mathbf{u}$ - $\mathbf{v}$ is tangent to the boundary

Occluded motion in the Fourier domain

$$\mathbf{F}(\omega,\omega_t) = \mathbf{A}(\omega)\delta(\mathbf{u}\cdot\omega+\omega_t) + \mathbf{B}(\omega)\delta(\mathbf{v}\cdot\omega+\omega_t) + \mathbf{C}(\omega,\omega_t)$$

$$\left|\mathbf{C}(\omega,\omega_t)\right| \leq d/(\mathbf{v}\cdot\omega+\omega_t) \text{ along the plane } \mathbf{u}\cdot\omega+\omega_t = c$$

The distortion $\mathbf{C}$ has hyperbolic decay



The hyperbolic decay of C has been first recognized by Beauchemin at al. for the particular case of straight line boundary

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