Precomputed Lighting: Theory and Practice

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Challenges in interactive rendering

- Generating realistic images interactively is hard
- Many dimensions of complexity
 - Geometric complexity
 - Material complexity
 - Meso-scale complexity
 - Lighting complexity
 - Transport complexity
 - Synergy
- This talk focuses on techniques that enable more lighting/transport complexity

Material Complexity

- Models how light interacts with a surface
 - Assume the "structure" of the material is below the visible scale
 - Simple variation
 - Twist maps



Meso-Scale Complexity

- Variations at a visible scale
 - not geometry
 - Bump/Roughness maps
 - Parallax Mapping/BTF's extreme examples of this



- What kind of lighting environment is an object in?
 - Directional/point lights
 - Directional + ambient
 - "Smooth" (low frequency) lighting
 - Completely general



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- How light interacts with objects/scene at a visible scale
 - Shadows
 - Inter-reflections
 - Caustics
 - Translucency (subsurface scattering)



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Some Of All Of This

- Real scenes have all of these forms of complexity
- Extreme realism on one is not necessarily that interesting
 - Incredible material models that are completely homogenous and lit by a single directional light
 - Great lighting environments for diffuse surfaces with no shadows

Precomputed Radiance Transfer (PRT)

- Models an object/scenes response to lighting expressed in a given basis
 - Can model arbitrary transport complexity
- Factors things into two steps
 - Off-line transport simulation that is independent of specific lighting environment
 - Simple run time component that depends on specific lighting environment



Rendering Equation

$$L(p \to \vec{d}) = L_e(p \to \vec{d}) + \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L(p \leftarrow \vec{s}) H_{N_p}(-\vec{s}) ds$$

Rendering Equation

$$L\left(p \to \vec{d}\right) = L_e\left(p \to \vec{d}\right) + \int_{\Omega} f_r\left(p, \vec{s} \to \vec{d}\right) L\left(p \leftarrow \vec{s}\right) H_{N_p}\left(-\vec{s}\right) ds$$



Radiance leaving point p in direction d

Rendering Equation

$$L(p \to \vec{d}) = \frac{L_e(p \to \vec{d})}{L_e(p \to \vec{d})} + \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L(p \leftarrow \vec{s}) H_{N_p}(-\vec{s}) ds$$



Radiance emitted from point p in direction d

Rendering Equation $L(p \to \vec{d}) = L_e(p \to \vec{d}) + \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L(p \leftarrow \vec{s}) H_{N_p}(-\vec{s}) ds$ Integral over directions s on the hemisphere around p



$$L(p \to \vec{d}) = L_e(p \to \vec{d}) + \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L(p \leftarrow \vec{s}) H_{N_p}(-\vec{s}) ds$$

BRDF at point *p* evaluated for incident direction *s* in outgoing direction *d*





$$L(p \to \vec{d}) = L_e(p \to \vec{d}) + \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L(p \leftarrow \vec{s}) H_{N_p}(-\vec{s}) ds$$



Lamberts law – cosine between normal and -s = dot(Np, -s)



$$L(p \rightarrow \vec{d}) = L_0(p \rightarrow \vec{d}) + L_1(p \rightarrow \vec{d}) + \cdots$$



Exit radiance expressed as infinite series







$$L(p \to \vec{d}) = L_0(p \to \vec{d}) + L_1(p \to \vec{d}) + \cdots$$
$$L_0(p \to \vec{d}) = \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L_s(p \leftarrow \vec{s}) V(p \to \vec{s}) H_{N_p}(-\vec{s}) ds$$



Visibility function - binary

$$L\left(p \to \vec{d}\right) = L_0\left(p \to \vec{d}\right) + L_1\left(p \to \vec{d}\right) + \cdots$$
$$L_0\left(p \to \vec{d}\right) = \int_{\Omega} f_r\left(p, \vec{s} \to \vec{d}\right) L_s\left(p \leftarrow \vec{s}\right) V\left(p \to \vec{s}\right) H_{N_p}\left(-\vec{s}\right) ds$$





$$L\left(p \to \vec{d}\right) = L_0\left(p \to \vec{d}\right) + L_1\left(p \to \vec{d}\right) + \cdots$$
$$L_1\left(p \to \vec{d}\right) = \int_{\Omega} f_r\left(p, \vec{s} \to \vec{d}\right) \frac{L_0\left(p \leftarrow \vec{s}\right)}{L_0\left(p \leftarrow \vec{s}\right)} \left(1 - V\left(p \to \vec{s}\right)\right) H_{N_p}\left(-\vec{s}\right) ds$$

All paths from source that take 1 bounce

$$L\left(p \to \vec{d}\right) = L_0\left(p \to \vec{d}\right) + L_1\left(p \to \vec{d}\right) + \cdots$$
$$L_i\left(p \to \vec{d}\right) = \int_{\Omega} f_r\left(p, \vec{s} \to \vec{d}\right) \frac{L_{i-1}\left(p \leftarrow \vec{s}\right)}{L_{i-1}\left(p \leftarrow \vec{s}\right)} \left(1 - V\left(p \to \vec{s}\right)\right) H_{N_p}\left(-\vec{s}\right) ds$$

All paths from source that take i bounces

Diffuse PRT

$$L\left(p \to \vec{d}\right) = L_0\left(p \to \vec{d}\right) + L_1\left(p \to \vec{d}\right) + \cdots$$
$$L_0\left(p \to \vec{d}\right) = \int_{\Omega} f_r\left(p, \vec{s} \to \vec{d}\right) L_s\left(p \leftarrow \vec{s}\right) V\left(p \to \vec{s}\right) H_{N_p}\left(-\vec{s}\right) ds$$

Diffuse PRT

$$L(p \to \vec{d}) = L_0(p \to \vec{d}) + L_1(p \to \vec{d}) + \cdots$$

$$\boxed{L_0(p \to \vec{d})} = \int_{\Omega} f_r(p, \vec{s} \to \vec{d}) L_s(p \leftarrow \vec{s}) V(p \to \vec{s}) H_{N_p}(-\vec{s}) ds$$

$$\boxed{L_0(p)} = \frac{\rho_d}{\pi} \int_{\Omega} L_s(-\vec{s}) V(p \to \vec{s}) H_{N_p}(-\vec{s}) ds$$

Diffuse PRT

$$L(p \to \vec{d}) = L_0(p \to \vec{d}) + L_1(p \to \vec{d}) + \cdots$$
$$L_0(p \to \vec{d}) = \int_{\Omega} \frac{f_r(p, \vec{s} \to \vec{d})}{p_r(p, \vec{s} \to \vec{d})} L_s(p \leftarrow \vec{s}) V(p \to \vec{s}) H_{N_p}(-\vec{s}) ds$$
$$L_0(p) = \frac{p_d}{\pi} \int_{\Omega} L_s(-\vec{s}) V(p \to \vec{s}) H_{N_p}(-\vec{s}) ds$$

Diffuse PRT

$$L(p \to \vec{d}) = L_0(p \to \vec{d}) + L_1(p \to \vec{d}) + \cdots$$
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$$L_0(p) = \frac{\rho_d}{\pi} \int_{\Omega} \frac{L_s(-\vec{s})}{L_s(p \to \vec{s})} V(p \to \vec{s}) H_{N_p}(-\vec{s}) ds$$






$$L_{0}(p) = \frac{\rho_{d}}{\pi} \int_{\Omega} \left(\sum_{i} l_{i} Y_{i}(-\vec{s}) \right) V(p \to \vec{s}) H_{N_{p}}(-\vec{s}) ds$$
$$L_{0}(p) = \frac{\rho_{d}}{\pi} \sum_{i} l_{i} \int_{\Omega} Y_{i}(-\vec{s}) V(p \to \vec{s}) H_{N_{p}}(-\vec{s}) ds$$

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$$L_{0}(p) = \frac{\rho_{d}}{\pi} \sum_{i} l_{i} \int_{\Omega} Y_{i}(-\vec{s}) V(p \rightarrow \vec{s}) H_{N_{p}}(-\vec{s}) ds$$
$$L_{0}(p) = \frac{\rho_{d}}{\pi} \sum_{i} l_{i} t_{pi}^{0}$$
$$L_{0}(p) = \sum_{i} l_{i} \frac{t_{pi}^{0}}{p_{i}}$$

$$L(p \to \vec{d}) = L_0(p \to \vec{d}) + L_1(p \to \vec{d}) + \cdots$$
$$L(p) = \sum_i l_i \left(t_{pi}^0 + t_{pi}^1 + \cdots \right)$$

$$L(p \to \vec{d}) = L_0(p \to \vec{d}) + L_1(p \to \vec{d}) + \cdots$$
$$L(p) = \sum_i l_i \left[\left(t_{pi}^0 + t_{pi}^1 + \cdots \right) \right]$$
$$L(p) = \sum_i l_i \left[t_{pi} \right]$$

Spherical Harmonics

- Can be used to represent signals over the sphere
 - General form is for complex signals, real form used for graphics
- Analogous to the Fourier basis
- Can be expressed as trigonometric functions of theta/phi
 - Mostly just useful for derivations, computing analytic formulas
- Also represented as polynomials of coordinates of a point on the unit sphere



SH Properties

- Basis functions are orthogonal
 - Integrate against themselves = 1
 - Integrate against other SH = 0
 - Makes projecting functions/signals simple
- Rotation invariance
 - Project(Rot(f)) = Rot(Project(f))
 - No aliasing as an object rotates, or lights rotate around an ojbect (ie: lights don't "wobble")
- Band limiting
 - Capture finer and finer frequencies as you add more terms (also helps with aliasing)





Precomputed Radiance Transfer (PRT)

- Compute objects response to a given number "light basis functions" off-line
 - Include arbitrarily complex light transport
- At run time rotate lighting into frame of the object
- Dot product of transfer vector and lighting vector generates exit radiance
- Limitations
 - Assumes rigid objects (see LDPRT later)
 - Lots of data (compression)
 - Distant lighting (see gradients, PRV later)



General Scene Response

- Rendering is scaling scene response by intensity of each "light" and summing
- Dot product generates an image
 - Factored form of an integral equation
- Relies on the spatial relationship between objects in the scene to be consistent (static scene)

Precomputed Basis

- Polynomial Texture Maps
 - [Malzbender2002]
 - Bi-quadratic polynomial (2 DOF)
- Steerable Illumination Textures
 - [Ashikhmin2002]
 - Steerable basis to model path of small area light (49 DOF)
- Directional Basis
 - [Hao2003]
 - Subsurface Scattering from directional lights
- Wavelets
 - [Ng03]
 - Models "all-frequencies"
 - Extended to glossy [Liu04,Ng04,Wang04]

Spatial Sampling Issues

• Relationship between spatial sampling densities over objects and light frequencies



Spatial Sampling Issues

- Light shrinks -> penumbra tightens
- Higher sampling density to "move" light over object/scene





Angular Sampling Issues

• Small lights clearly less



Sampling Issues

- Large (low frequency) lights
 - Coarse spatial sampling
 - Not a lot of storage
 - Large solid angles
 - Run time integration would be expensive
- Small (high frequency) lights
 - Fine spatial sampling
 - High storage
 - Small solid angles
 - Run time integration isn't that bad

Sampling Issues: Transport

- Bounced light is pretty much always low frequency
 - An illuminated wall is an area light
 - Even from a point (high frequency) light source
- Do you need to use high frequency basis to model high frequency inter-reflections???
 - Similar to duality discussed in [Ramamoorthi2001], interreflections are implicitly large area lights

Compression Goals

- Decode efficiently
 - As much on the GPU as possible
 - Render compressed representation directly
- Increase rendering performance
 - Big dot products can be expensive
- Reduce memory consumption
 - Not just on disk
- Not just for PRT, useful for any type of signal











CPCA

Compute a linear subspace in each cluster

$$\mathbf{M}_{p} \approx \tilde{\mathbf{M}}_{p} = \mathbf{M}_{C_{p}}^{0} + \sum_{i=1}^{N} w_{p}^{i} \mathbf{M}_{C_{p}}^{i}$$

CPCA

- Clusters with low dimensional affine models
- How should clustering be done?
- Static PCA
 - VQ, followed by one-time per-cluster PCA
 - optimizes for piecewise-constant reconstruction
- Iterative PCA
 - PCA in the inner loop, slower to compute
 - optimizes for piecewise-affine reconstruction

Static vs. Iterative



Related Work

- VQ+PCA [Kambhatla94] (static)
- VQPCA [Khambhatla97] (iterative)
- Mixture PC [Dony95] (iterative)
- Independently used with BTF's [Mueller03]
- More sophisticated models exist
 - [Brand03], [Roweis02]
 - Mapping to GPUs is challenging
 - Variable storage per vertex
 - Partitioning is more difficult (or requires more passes)
 - Worth investigating again on current GPU's like the Xbox360





How Compression Improves Rendering

Original expression

 $\boldsymbol{e}_p = \mathbf{T}_p \bullet \mathbf{L}$

Approximated with compression

$$e_{p} \approx \left(\mathbf{T}_{C_{p}}^{0} + \sum_{i=1}^{N} w_{p}^{i} \mathbf{T}_{C_{p}}^{i}\right) \bullet \mathbf{L}$$

- Looks more expensive
- "cluster" work is constant across vertices/texels

How Compression Improves Rendering

Factor, do highlighted portion on the CPU

$$e_{p} \approx \left(\mathbf{T}_{C_{p}}^{0} \bullet \mathbf{L}\right) + \sum_{i=1}^{N} w_{p}^{i} \left(\mathbf{T}_{C_{p}}^{i} \bullet \mathbf{L}\right)$$

- Rendering only depends on number of PCA vectors in a cluster
- Anything that is "linear" can be optimized this way
 - Combination of light maps, significantly reduce compute and storage, change intensities on the fly
PRT HLSL shader

```
float4 vAccumR = 0, vAccumG = 0, vAccumB = 0;
for (int i=0; i < (NUM_PCA/4); i++)
```

vAccumR += vPCAWeights[i] * aConsts[nOffset+1+(NUM_PCA/4)*0+i]; vAccumG += vPCAWeights[i] * aConsts[nOffset+1+(NUM_PCA/4)*1+i]; vAccumB += vPCAWeights[i] * aConsts[nOffset+1+(NUM_PCA/4)*2+i];

}

float4 vDiffuse = aConsts[nOffset]; vDiffuse.r += dot(vAccumR,1); vDiffuse.g += dot(vAccumG,1); vDiffuse.b += dot(vAccumB,1);

- Very lossy (4 PCA)
 - 11 instructions
 - NUM_CLUSTERS * 4 consts
 - 1 short4 + 1 byte per vertex
- Less lossy (12 PCA)
 - 17 instructions
 - NUM_CLUSTERS * 10 consts
 - 3 short4 + 1 byte per vertex
- DX SDK sample for details

What is a Transfer Vector?

- Each coefficient models how light expressed in the corresponding basis function contributes to exit radiance at a point
- Spherical function that integrates against distant lighting to compute exit radiance
 - Dot product generates exit radiance

Why Animating PRT is Hard

- Lighting and Transfer Vector have to be expressed in same coordinate system
 - For rigid objects just rotate the light once
 - For skinned characters you could compute rotated lighting at each bone and blend Rotating SH is expensive
- Trade off accuracy in transfer vector with efficient rotation...

Local Deformable PRT

- Irradiance Environment Maps are easy to rotate, but they don't model GI effects
- PRT Models complex GI effects, but hard to rotate
- We want something in between easy to rotate + handles some amount of GI effects

Local Deformable PRT

- Functions with circular symmetry around the Z axis project only into the Zonal Harmonic basis functions
 - These are the only class of functions that the SH convolution theorem can be applied to
- Evaluating basis functions in a direction (projection of a rotated delta function) generates a rotated form of the Zonal Harmonic basis functions (due to rotational invariance of SH)
- Given any circular symmetric function (in Z), rotating is just evaluating basis functions and scaling by ZH coefficients for given band



How Does LDPRT Work?

- Approximates transfer vector as a spherical function that has circular symmetry (around the shading normal) – zonal harmonic
- Just application of SH convolution theorem
- In between irradiance environment maps and PRT
- Shading normal from linear terms (direction of "maximal visibility")
- For a given shading normal, ZH coefficients that minimize squared error can be computed in closed form

Shading Normals

 Often used with ambient occlusion (sometimes called "bent normals")

How to use LDPRT?

- Can be used as a replacement for normal maps
 - No unique parameterization required
 - Works better for "local" surface properties
- Can be synthesized from scratch
 - Build leaf/skin textures for example
- Can be used instead of PRT
 - Less accurate transfer approximation



LDPRT Multiple Lobes

- One lobe only can represent circularly symmetric functions
 - Works ok for some textures, not as well for others
- Fit multiple lobes to approximate transfer vector
 - More data/reconstruction costs
 - Siggraph2005 paper
- Non-linear optimization problem
 - Used analytic gradients of objective function and BFGS
 - Fairly well behaved, but standard techniques are useful
 - Iteratively approximate a single lobe, compute residual, repeat
 - Multiple starts to avoid local minima
 - Any 3 lobe directions that aren't on a great circle can represent linears
 - >= 3 lobes, just explicitly store (easy to rotate linear)
 - 1 specific lobe (used in single lobe case) can represent as well

LDPRT Rendering

- Demo does full evaluation from previous slide (evaluates all basis functions in shading normal direction, scales each band by zonal harmonic coefficient...)
- Simple Optimization:

 $\sum_{l}\sum_{i\in l}z_{l}Y_{i}\left(\vec{N}\right)L_{i}$



LDPRT

- Light specialized rendering
 - Complexity O(n) instead of O(n²)
 - Fill a texture that contains dot(Y,L) for each band in each direction dynamically
 - 8x8 or 16x16 are probably doable with border textures
 - Without border textures another parameterization (Lat/Long in demo) works at 64x32
 - Use SIMD instructions to fill dynamic textures, do 4 texels at a time

Parameterized Models

- Can just fit to results of PRT simulation
- Can build an ad-hoc model that has intuitive parameters (build transfer vector on the fly)
- Can fit an intuitive model to simulation data

Simple Translucency

- Single degree of freedom (DOF)
 - "Optical Thickness", how much light bleeds through in the negative normal direction
 - Could be based on subsurface scattering simulation, demo is adhoc

Wrinkle Model

- Two DOF
 - Phase, position along canonical wrinkle

Wrinkle Model

- Two DOF
 - Phase, position along canonical wrinkle
 - Amplitude, max magnitude of wrinkle



Fit

- Compute several simulations
 - 64 discrete amplitudes
 - 255 unique points in phase
- Fit 32x32 textures
 - DC,Linear (4 numbers) using linear least squares
 - 3 lobes fit using non-linear least squares, all DOF fit at once (18k DOF about 5-10 min)
 - Quasi-Newton techniques don't work, approximate inverse Hessian is to large
 - Used non-linear conjugate gradients instead



Light flowing through space

- Lightmaps/PRT strictly deals with lighting response on surfaces
- What about in a volume?
- Needed to light objects moving through a scene that is modeled with transfer vectors

Representations for Lighting

- Plenoptic function [Adelson91]
- **F(x,y,z,θ,**φ,λ,**t)**
 - x,y,z : Position in space
 - θ, ϕ : Direction
 - $-\lambda$: Wavelength
 - t : Time
- Common to use 3 wavelengths (RGB)
- Time is a bit simplistic as the only means to model changes in illumination

Irradiance Volumes

- Approximation of Plenoptic Function convolved with normalized cosine kernel
- Irradiance Volumes [Greger98]
 - Original paper used "diffuse cube maps"
 - SH used in later papers
 - Static lighting, diffuse objects (no transport) scene not effected by objects

SH Gradients

- Model "mid range" illumination by using a Taylor expansion (in space) of projection into SH [Annen2004]
- This is a really good idea fairly inexpensive way to handle "local" lights
- Using N-gradient directions requires N times more work
 - 1 or 2 directional derivatives might make sense
- Also see Chris Oat's (ATI) GDC talk this year
 - But use compression!



Parameterized Radiance Volumes

- Extend Irradiance Volumes to handle dynamic lighting, render objects with PRT, hacks for how object effects lighting in scene
- Challenges
 - How to mix with SH Gradients
 - Compression



PRV Representation

- Use smooth (differentiable) BF so you can easily generate gradients
- Multi-level uniform quadratic b-splines
 - Not very expensive at run time
 - C1 continuous (gradients behave better)
 - Multi-level enables more aggressive compression
- K-nearest neighbor and radial basis functions also worth investigating
 - Cost/continuity a concern
 - Oct-trees used by Chris Oat as well



Generating a PRV

- Compute transfer matrices at a moderate density in space
- Fit coarse b-spline volume to transfer matrices
 - Linear least squares
- Compute residual, threshold
- Fit finer samples to residuals

PRV Optimizations

- Only encode matrices in a single "slice"
 - Chest height in a game
 - Encode derivative out of slice explicitly
- For finer scales, only encode luminance
 - Kind of like image/video compression
 - Lower angular frequency for chroma?
- PCA transfer matrices
 - Trade off PCA decoding with less interpolation

Final Thoughts

- PRT
 - Enables effects that are difficult with traditional techniques
 - Soft shadows from large area lights
 - Inter-reflections
 - Subsurface scattering
 - Easy to mix with traditional techniques
 - Split techniques based on light frequency (PRT for low, shadow maps for high)
 - Split based on transport path (PRT for indirect lighting, something else for direct)
 - Outdoor game
 - PRT for direct+indirect lighting from "skylight" (minus sun)
 - PRT for indirect lighting from sun
 - Conventional techniques for direct lighting from sun

Final Thoughts

- LDPRT
 - Can be used for surface details
 - Trivial to skin/deform (but shadows in tangent space or rest configuration)
- PRV
 - Tying objects into the lighting used for the scene is a good idea, and done already in games
 - Parameterizing lighting makes sense going forward
 - Independent light maps
 - Outdoor lighting (sky light model)
- Compression
 - Always use with PRT
 - Worth using for other scenarios (multiple light maps in particular)

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